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A Survey of Neutron and Proton Binding Energies

By N. FEATHER, F.R.S.
The University, Edinburgh

SUMMARY

Limiting values of last-nucleon binding energies are available from measured reaction energies of induced transformations satisfying the selection rule $\Delta A=0$, ± 1 (A=mass number) in about 140 cases. In 14 cases upper and lower limiting values are known separately, and these agree in each case within the limits of experimental error. Taking the other 126 values as ‘ true ’ (rather than limiting) values, and using information concerning total energies of β-disintegration, the number of last-nucleon binding energies which are reasonably well determined is increased to more than 600. These values are tabulated, and discussed in relation to the v. Weizsäcker mass formula. Apart from certain anomalies which are treated individually, the discussion brings out the effect of α-unit structure in light nuclei, diminishing in importance to become almost negligible beyond $A=40$, and yields mean values for nucleon-parity dependent terms in the range $A\sim 215$. The recovery of ‘ normal ’ last-nucleon binding energy after completion of a closed shell is followed in detail over the range $126\leqslant N\leqslant 136$ (N=neutron number) and somewhat less closely over the range $82\leqslant Z\leqslant 92$ (Z=proton number).

IN principle, $B_n\binom{A}{Z}$, the energy of binding of the last-added neutron in the nucleus $\binom{A}{Z}$, may be determined directly from a study of the γ-radiation emitted in the neutron-capture process

$$\binom{A-1}{Z}(n,\gamma)\binom{A}{Z}. \qquad (1)$$

For this determination to be practicable the next-lower isotope $\binom{A-1}{Z}$ must be a stable, or long-lived and available in sufficient amount ; then, if Q_1 is the reaction energy,

$$B_n\binom{A}{Z}=Q_1. \qquad (1\,a)$$

Experimentally, it should be noted, Q_1 is not unambiguously determinable, but ${}_1E_\gamma$ the highest quantum energy in the capture γ-ray spectrum is : from this point of view (1 *a*) should rather be written

$$B_n\binom{A}{Z} \geqslant {}_1E_\gamma . \qquad (1\,b)$$

When the species $\binom{A}{Z}$ it itself available in sufficient amount, $B_n\binom{A}{Z}$ may be determined directly from the threshold energy for the reaction

$$\binom{A}{Z}(\gamma, n)\binom{A-1}{Z}. \qquad (2)$$

If Q_2 is this threshold energy, obviously

$$B_n\binom{A}{Z} = Q_2 . \qquad (2\,a)$$

Here again, Q_2, which refers to the ground-to-ground state transition, is not necessarily determinable ; the final nucleus $\binom{A-1}{Z}$ may ' always ' be produced in an excited state by the photodisintegration process. Admitting this possibility, if the effective threshold occurs with γ-radiation of quantum energy ${}_2E_\gamma$, then, more cautiously, we have

$$B_n\binom{A}{Z} \leqslant {}_2E_\gamma . \qquad (2\,b)$$

In some cases (chiefly when Z is even) reactions (1) and (2) may be applied in the same determination. If, in such a case, ${}_1E_\gamma = {}_2E_\gamma$, clearly $B_n\binom{A}{Z}$ is known without serious doubt (see, for example, Huizenga, Magnusson, Simpson and Winslow 1950).

Corresponding to reactions (1) and (2), the deuteron-induced reactions

$$\binom{A-1}{Z}(d, p)\binom{A}{Z} \qquad (3)$$

$$\binom{A}{Z}(d, t)\binom{A-1}{Z} \qquad (4)$$

provide only slightly less direct methods of determining $B_n\binom{A}{Z}$. If Q_3, Q_4 are the ground-to-ground state reaction energies, we have

$$B_n\binom{A}{Z} = Q_3 + B_n\binom{2}{1} \qquad (3\,a)$$

$$B_n\binom{A}{Z} = B_n\binom{3}{1} - Q_4 . \qquad (4\,a)$$

For these determinations $B_n\binom{2}{1}$ is known on the basis of (1) and $B_n\binom{3}{1}$ on the basis of (3). As before, experiment gives, not Q_3 or Q_4 directly, but only lower limiting values of these quantities (ground-to-ground state transitions may be ' unobserved ') ; then, as previously, the reaction involving neutron addition to $\binom{A-1}{Z}$ leads to a lower and that involving neutron removal from $\binom{A}{Z}$ to an upper limit for $B_n\binom{A}{Z}$. In this particular the reaction

$$\binom{A-1}{Z}(t,\, d)\binom{A}{Z} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \qquad (5)$$

is obviously equivalent to (3), and the reactions

$$\left.\begin{array}{l} \binom{A}{Z}(p,\, d)\binom{A-1}{Z} \\ \binom{A}{Z}({}^3\mathrm{He},\, \alpha)\binom{A-1}{Z} \end{array}\right\} \quad . \; . \; . \; . \; . \; . \qquad (6)$$

are equivalent to (4). On the basis of one or more of reactions (1) to (6) in each case, and as a result of the combined work of many investigators, information regarding the energy of binding of the last neutron in some 135 species is now available which is not available with equal precision from other sources.

In the matter of last-proton binding energies the position is in almost every way less favourable. Reactions corresponding to the neutron-transfer reactions (1) to (6) may be used for proton transfer, but experimentally they are intrinsically more difficult to investigate than their prototypes, as brief consideration will show. All of them are affected by ‘ potential barrier ’ effects. Thus, corresponding to (1), the reaction $(p,\, \gamma)$ cannot be studied with thermal protons, and for the evaluation of Q an additional energy determination is required (though the ‘ incident ’ energy may be well-defined if a suitably isolated ‘ resonance ’ is employed). Corresponding to (2), the threshold is less abrupt for proton than it is for neutron emission because of ‘ barrier penetration ’. Corresponding to (3), neutron energy spectrometry is technically at least one degree more difficult than proton energy spectrometry. Reactions (4) and (5), even in the neutron case, are of very restricted applicability, and the corresponding proton-transfer reactions $(d,\, {}^3\mathrm{He})$, $({}^3\mathrm{He},\, d)$ are even less useful. Not much use can be made, either, of the reactions $(n,\, d)$, $(t,\, \alpha)$, corresponding to (6). In total, information regarding the energy of binding of the last proton in some five species is all that is at present available from this mode of approach, at least if better accuracy is sought than is otherwise attainable.

Using induced reaction energies there is, however, one other mode of approach to a knowledge of the energies of binding of last protons which may sometimes be employed. If Q_7 is the ground-to-ground state reaction energy for the nucleon-exchange reaction

$$\binom{A-1}{Z-1}(p,\, n)\binom{A-1}{Z} \quad . \; . \; . \; . \; . \; . \; . \; . \qquad (7)$$

clearly

$$Q_7 = B_p\binom{A}{Z} - B_n\binom{A}{Z} \quad . \; . \; . \; . \; . \; . \; . \qquad (7\,a)$$

and a simple identity (Feather 1952 b) also gives

$$Q_7 = B_p\binom{A-1}{Z} - B_n\binom{A-1}{Z-1}. \quad . \quad . \quad . \quad . \quad (7\,b)$$

In this case, if $B_n\binom{A}{Z}$ or $B_n\binom{A-1}{Z-1}$ is known, a determination of Q_7 allows $B_p\binom{A}{Z}$ or $B_p\binom{A-1}{Z}$ to be calculated. Experimentally Q_7 is determined from the threshold energy of the (p, n) reaction (7), and, although this reaction is conditioned by ‘ barrier penetration ’ effects, the determination can be made with considerable accuracy. Neutrons of near-thermal energy can be detected with high efficiency, and reliable methods of ‘ extrapolation to threshold ’ have been devised. As in all cases, the major residual uncertainty is as to whether or not the ground-to-ground state transition has been observed. Allowing for the possibility that in any instance it has not, the experimental threshold merely fixes a lower limit to Q_7, with results for further calculations sufficiently indicated by (7 *a*).

In a small number of cases the reaction which is the reverse of (7), namely

$$\binom{A-1}{Z}(n, p)\binom{A-1}{Z-1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

is exothermic for stable target species. In such cases the reaction energy Q_8 may be determined in experiments using thermal neutrons (otherwise the determination is technically difficult, and usually of low accuracy). Obviously

$$Q_8 = B_n\binom{A}{Z} - B_p\binom{A}{Z} = B_n\binom{A-1}{Z-1} - B_p\binom{A-1}{Z} \quad . \quad . \quad (8\,a)$$

and ‘ good ’ experimental values of Q_8 may be used for present purposes exactly as those of Q_7.

So far as the target nucleus is concerned, the nucleon-exchange reactions (p, n) and (n, p) are equivalent to negative β-emission and the electron-capture transformation, respectively. Disintegration energies for these processes may therefore be used for all purposes for which Q_7 and Q_8 might be employed, and because so much information is available in this category their use will forthwith be considered in some detail.

First it may be remarked that no lack of generality will result if we write throughout of negative β-disintegration alone. An electron capture process $\binom{A}{Z+1} \rightarrow \binom{A}{Z}$, of total transformation energy E, is formally equivalent to the reverse (negative β-disintegration) process $\binom{A}{Z} \rightarrow \binom{A}{Z+1}$ characterized by disintegration energy $-E$. Admitting negative as well as positive values of $T_\beta^-\binom{A}{Z}$, therefore, we may regard both types of process as in a single category. Here, as in previous notes (Feather

1952 b, 1953), $T_\beta^-\binom{A}{Z}$ represents the ground-to-ground state transition energy—and it may be remarked that determination of this quantity in practice frequently requires energy measurements for both β- and γ-radiations, and correlation studies involving the two.

Now let us suppose that, for a single species $\binom{A}{Z}$, $B_n\binom{A}{Z}$ is known from a determination based upon one of the reactions (1) to (6). If $T_\beta^-\binom{A}{Z}$ is also known, we have

$$B_p\binom{A}{Z+1}=B_n\binom{A}{Z}+T_\beta^-\binom{A}{Z}-T_\beta^-\binom{1}{0}-\epsilon\binom{Z+1}{Z} \quad . \quad . \quad (9)$$

where $\epsilon\binom{Z+1}{Z}$ is the energy of ‘ adiabatic ’ reorganization of the extra-nuclear electrons which is liberated in the β-process. This reorganization energy is of order of magnitude $1{\cdot}5\times10^{-4}Z$ Mev (Allard 1948, Foldy 1951, Scott 1952). It has not been taken into consideration in eqns. (1 *a*) to (8 *a*), but is included here because the possibility of cumulative error is in question. For we are in fact assuming that, besides $T_\beta^-\binom{A}{Z}$, $T_\beta^-\binom{A-1}{Z}$, $T_\beta^-\binom{A}{Z+1}$, $T_\beta^-\binom{A-1}{Z+1}$, . . . are known. Then following (9) we have

$$\left.\begin{aligned}
B_n\binom{A}{Z+1}&=B_p\binom{A}{Z+1}-T_\beta^-\binom{A-1}{Z}+T_\beta^-\binom{1}{0}+\epsilon\binom{Z+1}{Z}\\
B_p\binom{A}{Z+2}&=B_n\binom{A}{Z+1}+T_\beta^-\binom{A}{Z+1}-T_\beta^-\binom{1}{0}-\epsilon\binom{Z+2}{Z+1}\\
B_n\binom{A}{Z+2}&=B_p\binom{A}{Z+2}-T_\beta^-\binom{A-1}{Z+1}+T_\beta^-\binom{1}{0}+\epsilon\binom{Z+2}{Z+1}\\
&\cdots\cdots\cdots\cdots\cdots\cdots
\end{aligned}\right\} \quad . \quad (10)$$

In the sequence of equations represented by (9) and (10) it will be observed, incidentally, that successive members correspond to (7 *b*) and (7 *a*), respectively, and it will further be observed that no cumulative error results from the neglect of the small quantities ϵ at each stage—or from the repeated inclusion of the error attaching to the neutron β-disintegration energy $T_\beta^-\binom{1}{0}$—since these quantities continually cancel as the sequence proceeds. Formally, we may say that if we know the energy of negative β-disintegration for each member of the two neighbouring isobaric series

$$\binom{A}{Z},\ \binom{A}{Z+1},\ \binom{A}{Z+2},\ \cdots$$
$$\binom{A-1}{Z},\ \binom{A-1}{Z+1},\ \binom{A-1}{Z+2},\ \cdots,$$

and the energy of binding of the last neutron in the ‘ key ’ nucleus $\binom{A}{Z}$, we may calculate the energies of binding of the last proton and the last neutron in each of the isobaric nuclei

$$\binom{A}{Z+1}, \binom{A}{Z+2}, \binom{A}{Z+3} \cdots.$$

Regarding the first equation of (10), rather than (9), as beginning the sequence, we can obviously make a similar statement which assumes as initial datum the energy of binding of the last proton in $\binom{A}{Z+1}$ (determined, for example, on the basis of reaction (7) or (8)) rather than of the last neutron in $\binom{A}{Z}$. These two complementary statements, it will now be observed, show under what conditions the last-neutron and last-proton binding energies for each member of a series of isobaric nuclei of progressively decreasing neutron excess may be determined, if the last-neutron (or-proton) binding energy is known for the key (first) nucleus of the series. Clearly there is nothing fundamental in the choice of direction for the process of step-wise calculation involved in such determinations. Reversing the order of the sequence of equations represented by (9) and (10)—and the roles of ‘ known ’ and ‘ to be calculated ’ binding energies entering in each equation—it is evident that last-neutron and last-proton binding energies for successive members of an isobaric series of progressively increasing (rather than decreasing) neutron excess may similarly be calculated in terms of the last-neutron (or -proton) binding energy of a key species and the relevant energies of negative β-disintegration.

Since in fact our key species is usually a stable species, we shall obviously require to use the sequence of eqns. (9) and (10) both in the direct and also in the reverse sense in order to make full use of the information available.

For the heaviest elements ($Z>82$) energies of α-disintegration also enter as primary experimental data for the purpose of our survey. As is well recognized, if $T_{\beta}^{-}\binom{A}{Z}$ is available from direct experiment, and if the energies of α-disintegration of the species

$$\binom{A+4}{Z+2}, \binom{A+8}{Z+4}, \binom{A+12}{Z+6} \cdots.$$

$$\binom{A+4}{Z+3}, \binom{A+8}{Z+5}, \binom{A+12}{Z+7} \cdots.$$

are known, energies of β-disintegration for the species

$$\binom{A+4}{Z+2}, \binom{A+8}{Z+4}, \binom{A+12}{Z+6} \cdots.$$

may be calculated by the method of closed cycles, though it may be quite impracticable to determine them directly. And a similar formalism obviously represents the case in which the known energy of β-disintegration is that of the heaviest rather than the lightest member of a series of constant neutron excess (constant isotopic number, D), and 4 units common difference in mass number A, when the method is applied in the sense of A decreasing rather than the reverse. Energies of β-disintegration, calculated in this way on the basis of α-disintegration energies, may be used in eqns. (9) and (10) as before.

By use of eqns. (7 *a*), (7 *b*), (8 *a*), (9) and (10), the total number of last-nucleon binding energies concerning which information is directly available, or has been deduced from the conservation law, has been brought up to well over 600. The main object of this paper is to present these numerical results and to comment on them. The results themselves are given in the table in Appendix I, and certain explanatory notes concerning matters of detail are collected in Appendix II. Here it may be well to make a few additional remarks of a general nature, before embarking on the formal discussion.

In the first place it should be remarked that probable errors are assigned to all binding energies given in the table. Usually these have been assigned on the basis of the values attached by the original investigators to the various experimental quantities used in the calculations. Where such values are not given in the literature probable errors have been assigned somewhat arbitrarily, taking into account in each case the experimental method involved, the strength of source employed, and similar factors when known. Probable errors as quoted take no account of possible mistakes in interpretation. In this connection—and this is the second point to be made—it has been assumed, with numerically insignificant exceptions (to which attention is drawn in Appendix II), that the 'maximum' reaction energies obtained from experiment and used in the calculations are in fact the energies of ground-to-ground state transitions in all cases. Whilst it would represent extreme good fortune if this assumption, with the very few exceptions already recognized, were to prove uniformly correct in the upshot, it may be pointed out that upper and lower limiting values of last-neutron binding energy are available, and have been used combined in the table, for each of the following species, namely,

$$^{2}\mathrm{H},\ ^{11}\mathrm{B},\ ^{13}\mathrm{C},\ ^{25}\mathrm{Mg},\ ^{26}\mathrm{Mg},\ ^{29}\mathrm{Si},\ ^{57}\mathrm{Fe},\ ^{87}\mathrm{Sr},\ ^{91}\mathrm{Zr},\ ^{118}\mathrm{Sn},\ ^{195}\mathrm{Pt},$$
$$^{207}\mathrm{Pb},\ ^{208}\mathrm{Pb}\ \text{and}\ ^{238}\mathrm{U},$$

and that these upper and lower limiting values coincide without significant discrepancy in each case. This is a heartening result which inspires substantial confidence in the general assumption—see, however, Appendix II for the case of $B_n\begin{pmatrix}210\\83\end{pmatrix}$.

The third point to be made is that occasionally values are given in the table which are themselves lower (or upper) limiting values. These are additional to the 600 and more values which are given precisely, and they are chosen from a much greater number, which might similarly have been given as upper or lower limits, as belonging to those species for which the precise values are likely to be fairly close to the limiting values quoted. In such cases knowledge of the limiting values may contribute usefully to an overall picture of trends such as our discussion is intended to provide.

Finally, it should be pointed out that the values of binding energy given in the table do not depend significantly upon direct determinations of nuclear mass. Exceptions to this statement are few enough to be mentioned here. Mass determinations are involved, along with the β-disintegration energy of the neutron (or the $n-p$ difference calculated from this and other disintegration data), in providing the best values of $B_n\binom{4}{2}$ and $B_p\binom{4}{2}$, and the same mass determinations (essentially those of the proton and the α-particle (Bainbridge 1951)) have been used in calculating $B_n\binom{210}{83}$ (see Appendix II). For the obviously unstable naturally-occurring species $^{50}_{23}\mathrm{V}$ mass determinations (Johnson 1952) at present provide our only information regarding the extent of its instability. These, therefore, have been used. Otherwise the fact that disintegration energies are known with an accuracy which is tolerably constant over the whole range of A from 1 to 248, whereas the absolute (as distinct from the relative) accuracy of exact masses is still in general considerably less for heavier than for lighter atoms, has led us to base our calculations almost entirely on the former determinations. And, so far as induced reaction energies are concerned, it will be observed that we have left out of account all reactions in which α-particles are emitted or absorbed, except those of the types (^{3}He, α) and (t, α). Except for these two types (and the reverse processes), reactions in which α-particles are involved are characterized in each case by the transfer of more than a single nucleon from incident particle to final nucleus. In such an event the reaction energy cannot be expressed in terms of the difference between last-nucleon binding energies appropriate to projectile and product, and the information provided for our purposes is correspondingly indirect. That, by restricting our basic evidence to the information provided by induced and spontaneous transformations for which $A=0, \pm 1$ (which is effectively our selection rule), we have been able to compile a table of binding energies as extensive as that given in Appendix I is a clear indication of the amount of quantitative information which awaits the empiricist in these days. By following our selection rule we have clearly forfeited the opportunity of many corrective checks on individual energy values, but we have thereby been able to present a strictly homogeneous body of evidence for detailed discussion. We proceed now to that discussion without further preliminary.

Of the various nuclear models, the liquid-drop model, as represented by the semi-empirical mass formula of v. Weizsäcker, provides the most direct basis for numerical computation. As many previous writers have done, we employ it here—at least for the initial phases of our discussion.

Being to a large extent a non-particulate 'featureless' model, the liquid-drop model constitutes a norm against which the particulate features (e.g those of shell structure) of real nuclei should appear as discrepancies between prediction and fact, if these are suitably set the one against the other. Because our main aim will be to attempt to elicit the particulate features of the structure of real nuclei, we shall start from the v. Weizsäcker formula in its simplest form, not including in it initially any spin-dependent or nucleon-parity dependent term (see Saha and Saha 1946, Bohr and Wheeler 1939), or any term representative of the compressibility of nuclear matter (Feenberg 1947). We write, then, for $N\binom{A}{Z}$, the 'exact' mass of the nucleus $\binom{A}{Z}$, simply

$$N\binom{A}{Z} = ZN\binom{1}{1} + (A-Z)\,N\binom{1}{0} - \alpha A + \beta\frac{(A-2Z)^2}{A} + \gamma A^{2/3} + \delta\frac{Z(Z-1)}{A^{1/3}} \quad . \; . \; . \; . \; . \quad (11)$$

We may refer to (11) as the four-constant v. Weizsäcker mass formula.

Obviously, the most straightforward method of using (11) for our comparisons would be to obtain expressions for $B_n\binom{A}{Z}$ and $B_p\binom{A}{Z}$ by use of this formula, to insert accepted values of the constants α, β, γ, δ, and to compare predicted and experimental values of binding energies on this basis. This we shall not do at all systematically, but merely as an example of its use we shall follow the method in respect of neutron binding energies for three series of species of large A, those of constant isotopic number $D=44$, $D=45$ and $D=46$. Other things being equal, the liquid-drop model might be expected to represent real nuclei most closely when A is large.

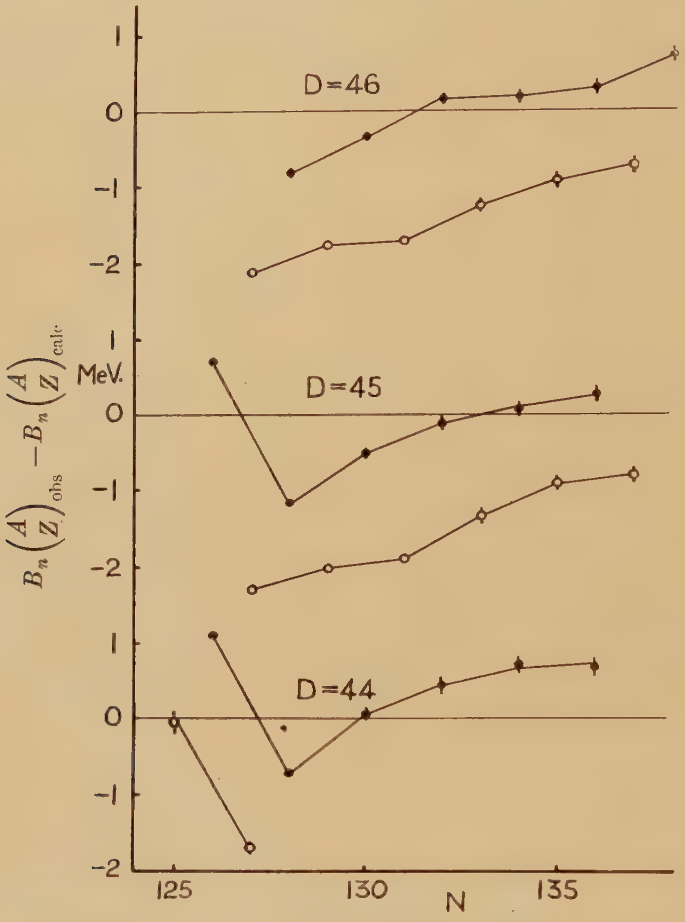
From (11) we have, to sufficient accuracy,

$$B_n\binom{A}{Z} = \alpha - \beta\left(1 - \frac{4Z^2}{A(A-1)}\right) - \frac{2}{3}\gamma\frac{1}{A^{1/3}} + \frac{1}{3}\delta\frac{Z(Z-1)}{A^{4/3}} \quad . \; . \; . \quad (12)$$

and for accepted values of the constants we may write $\alpha = 14{\cdot}4$ MeV, $\beta = 20{\cdot}0$ MeV, $\gamma = 15{\cdot}0$ MeV, $\delta = 0{\cdot}60$ MeV (see Pryce 1950). With these values, the first two terms in (12) are of prime importance in determining the absolute magnitude of $B_n\binom{A}{Z}$—the third and fourth terms cancelling one another, to a very crude approximation, for $D \sim 45$—and the second term is of chief importance (though the third and fourth terms are not negligible, and contribute in the same sense) in respect of the variation of $B_n\binom{A}{Z}$ with A and Z (D constant) in this range. Figure 1 shows how the excess of observed over calculated binding energy varies with neutron number N in the range in question.

We shall return to fig. 1 presently, meanwhile we note that the v. Weizsäcker expression for $B_n\binom{A}{Z}-B_p\binom{A}{Z}$ is considerably simpler than (12), and that in fact this difference of last-nucleon binding energies is more directly observable experimentally (when unstable species are in

Fig. 1



question) than is $B_n\binom{A}{Z}$ or $B_p\binom{A}{Z}$ individually. As has been pointed out already (Feather 1952 b)

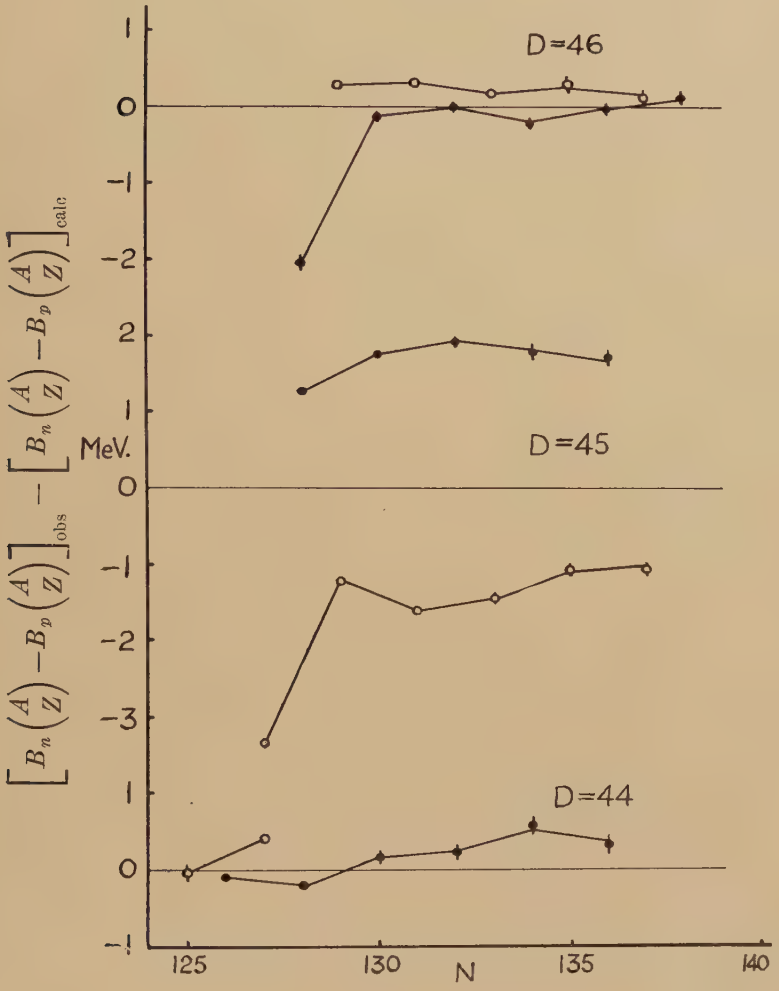
$$B_n\binom{A}{Z}-B_p\binom{A}{Z}=T_\beta^-\binom{1}{0}-T_\beta^-\binom{A-1}{Z-1}, \quad . \quad . \quad . \quad (13)$$

electronic reorganization energy being neglected (see (10)). Also, on the basis of (11), without further approximation,

$$B_n\binom{A}{Z}-B_p\binom{A}{Z}=-4\beta\frac{A-2Z}{A-1}+2\delta\frac{Z-1}{(A-1)^{1/3}}. \quad . \quad . \quad (14)$$

The two equations represent determinations of 'observed' and 'calculated' values of $B_n\binom{A}{Z} - B_p\binom{A}{Z}$: these we shall now compare. The excess of observed over calculated neutron/proton binding energy difference is plotted against N in fig. 2, again for the three series of species

Fig. 2



for which $D=44$, 45 and 46, and using the same values of β and δ as previously employed. On this basis, and over this range of D, $B_n\binom{A}{Z} - B_p\binom{A}{Z}$, calculated from (14) as of the order of 1 MeV, is given by the difference of two large terms each of the order of 17 MeV. These terms contribute about equally and in the same sense to the variation of

$B_n\binom{A}{Z} - B_p\binom{A}{Z}$ with A and Z (D constant) in this range. We now discuss figs. 1 and 2 together.

The first point of interest which arises from fig. 1 is the evidence which it provides of the need of a nucleon-parity dependent term in the mass formula. Assuming that the four-constant formula (11) applies without such addition only when both N and Z are even, a little consideration will show that we should expect $B_n\binom{A}{Z}$ to be greater than is given by (12) when N is even and correspondingly less than is given by (12) when N is odd (independently of the parity of Z). This is precisely the behaviour indicated by fig. 1. For each of the three series of nuclei represented on the figure, plotted points fall clearly into two groups, the upper group characterized by even N, the lower by odd N. Moreover, for $D=46$ and $D=44$ the upper groups of points represent nuclei of even Z the lower nuclei of odd Z, whilst for $D=45$ the reverse obtains. Thus our parenthesis 'independently of the parity of Z' is fully substantiated. As the figure shows, the magnitude of the required nucleon-parity term is not constant: over a range of N of 10 units ($N \sim 130$) it exhibits obvious variations which are certainly significant.

In more detail, if the parity-dependent term in (11) is represented by $\chi(n)$ when N is odd and Z even, by $\chi(p)$ when Z is odd and N even, and by $\chi(np)$ when both N and Z are odd, the additional term in (12) is $\chi(n)$, $\chi(p)-\chi(np)$, $\chi(np)-\chi(p)$, $-\chi(n)$ according as the neutron–proton parity type of the nucleus $\binom{A}{Z}$ is $\binom{e}{e}$, $\binom{o}{o}$, $\binom{e}{o}$ or $\binom{o}{e}$. Here it should be noted that the parity types $\binom{e}{e}$, $\binom{o}{o}$ belong to nuclei with D even and the types $\binom{e}{o}$, $\binom{o}{e}$ to nuclei with D odd—and it should further be noted that our notation does not allow for the variations of χ with N to which attention has just been drawn. This last is a real limitation; if we accept it, however, using average values, it follows that the separation of the two branches of each 'curve' for D constant on fig. 1 (effectively either the 'curve' for $D=45$ or that for $D=46$), is a measure of $\chi(np)+\chi(n)-\chi(p)$. Values of 1·27 Mev and 1·33 Mev are obtained for this quantity taking averages over the range $128 \leqslant N \leqslant 136$ for $D=45$ and over the range $128 \leqslant N \leqslant 137$ for $D=46$.

The second significant feature of fig. 1 concerns the effect of shell closure at $N=126$, which is represented in one way or another by the lie of the points belonging to each of the six groups of nuclei concerned. The abrupt decrease in binding energy following the completion of the closed shell of 126 neutrons is shown both for the $\binom{e}{e}$ and the $\binom{o}{o}$ nuclei having $D=44$, and also for the $\binom{e}{o}$ nuclei having $D=45$, and the slow recovery of normal values is evidenced by the trends indicated for each of the parity groups both for $D=46$ and $D=45$, and for the group of $\binom{e}{e}$ nuclei having $D=44$, for which alone (for this value of D) experimental values are available. Closer inspection shows that the abrupt decreases (whether as between $N=126$ and $N=128$, or between $N=125$ and $N=127$) are the same, in the three cases represented, within the limits of experimental error, being $1\cdot81\pm0\cdot03$, $1\cdot65\pm0\cdot16$ and $1\cdot86\pm0\cdot05$ Mev in the order above given, although the features of recovery of normal binding energy are manifestly different for even- and odd-numbered neutrons. Thus for odd N the trends exhibited in the figure for $D=45$ and $D=46$ are very closely parallel, showing the main recovery delayed until $N>131$, but they are distinct from the trends characteristic of even N (particularly the closely parallel trends for $D=44$ and $D=45$) which are of the more familiar 'exponential' form with the greatest degree of recovery in the first step. On the other hand, if (12), with accepted values of constants, provides a satisfactory norm, we have to conclude, for the more rapid recovery when N is even, that this recovery is not fully complete even at $N=136$, that is when 10 additional neutrons have been added outside the closed shell.

The only other comment on fig. 1 necessary here is that the position of the zero of ordinates for each curve is very sensitive to our choice of values for the v. Weizsäcker constants α and β (see above). The choice we have made appears in the upshot to have been a satisfactory one, but the essence of our comments would have been the same, even if the details of that choice had been different (within reasonable limits).

Turning to fig. 2 we note at once the obvious difference in the general aspect of the 'curves' for even and odd D, as represented on the figure. We note also an effect, due presumably to shell closure, which appears for $D=45$ and $D=46$ but not for $D=44$, although the curve for this value of D covers the range of N in which such an effect might, on first consideration, have been expected to show. The second of these observations has already been noted, and satisfactorily explained (Feather 1952 a, 1952 b): for $D=44$, $N=126+m$, $Z=82+m$, m integral, and, in order to explain the effect, we have only to suppose (or conclude) that the 'shell-closure sensitive' term in the expression for $B_p\binom{A}{Z}$ exactly parallels the corresponding term in $B_n\binom{A}{Z}$ as m varies from -1 to $+10$—or if the upwards trend of the 'N even' branch of the curve for $D=44$ is regarded as significant, that the recovery of normal binding energy by nucleons of the new shell is slightly less rapid for even-numbered protons added beyond $Z=82$ than for even-numbered neutrons added beyond $N=126$ (at least in nuclei for which $D=44$). This having been said, we interpret the presumed effects of shell closure exhibited between $N=127$ and $N=129$ on the curve for $D=45$ and between $N=128$ and $N=130$ on the curve for $D=46$—and previously mentioned—as representing essentially the decrease in proton binding energy between $Z=82$ and $Z=84$ in each case. The magnitude of this decrease is obtained, by combining the data of figs. 1 and 2, as $1{\cdot}84\pm0{\cdot}08$ and $1{\cdot}39\pm0{\cdot}15$ MeV in nuclei for which $D=45$, $D=46$, respectively.

As in fig. 1, so in fig. 2, the curve for each value of D falls naturally into two branches. But on fig. 2 the separation of the branches is very much greater for odd D ($D=45$) than for even D ($D=44$, 46), and the order of the branches alternates with the parity of D, the upper branch corresponding to N even when D is odd, and to N odd when D is even. In both these particulars the general aspect of fig. 2 is different from that of fig. 1. This is the first observation mentioned above: as will appear, it can be explained satisfactorily in terms of the nucleon-parity terms, $\chi(np)$, $\chi(n)$ and $\chi(p)$, already introduced. Taking appropriate averages, as before, we obtain $2\chi(np)$ as the mean separation of the two branches of the curve for an odd value of D, and $2(\chi(p)-\chi(n))$ as the mean separation of the two branches for an even value of D, on fig. 2. For $D=45$ and $129\leqslant N\leqslant136$ the mean separation is $3{\cdot}11$ MeV, thus we obtain, as an average value over this range, $\chi(np)=1{\cdot}56$ MeV. Taking this together with the average value of $\chi(np)-\chi(n)-\chi(p)$ for $D=45$ deduced from fig. 1, we have $\chi(p)-\chi(n)=0{\cdot}29$ MeV—which is entirely consistent with the value for this quantity as deduced from the two branches of the curve for $D=46$ on fig. 2.

Summarizing briefly at this interim stage, we may note that for the range $D\sim45$, $N\sim130$ we have obtained 'experimental' mean values for the quantities

$$\left.\begin{array}{l}\chi(np)+\chi(n)-\chi(p)\\ \chi(np)\\ \chi(p)-\chi(n)\end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

—and that the values so obtained have proved self-consistent. We have based these derivations on the assumption that the v. Weizsäcker four-constant mass formula (11), with nucleon-parity dependent terms $\chi(n)$, $\chi(p)$ and $\chi(np)$ added as appropriate, and with a 'shell-closure sensitive' term included, is a sufficiently good representation of fact. Still following this assumption we may derive an experimental mean value for

$$\chi(np)-\chi(n)-\chi(p) \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

from a comparison of observed and calculated values of

$$B_n\binom{A}{Z}-B_n\binom{A-1}{Z-1}.$$

According to (12), based on the four-constant mass formula without additional terms, and using accepted values of the constants, $B_n\binom{A}{Z}-B_n\binom{A-1}{Z-1}$ should remain steady at $0{\cdot}21\pm0{\cdot}01$ MeV for $A-2Z=45$ over the whole range $127\leqslant N\leqslant137$. In actual fact this difference between the binding energies of the Nth neutron in corresponding nuclei of the sequences $D=45$, $D=46$, though very nearly constant as N varies (extreme values $+0{\cdot}19$ MeV and $-0{\cdot}14$ MeV), has mean value $0{\cdot}02\pm0{\cdot}03$ MeV rather than $0{\cdot}21$ MeV. When the nucleon-parity dependent terms are included, the excess of the 'observed' over the 'calculated' mean value is found to be given by (16) (see below). We have, then, $\chi(np)-\chi(n)-\chi(p)=-0{\cdot}19\pm0{\cdot}03$ MeV, for the series in question. Combining this with the values already deduced for the quantities specified in (15), we have, finally, as significant average values, for $D=45$,

$$\chi(p)=1{\cdot}02 \text{ MeV}$$

$$\chi(n)=0{\cdot}73 \text{ MeV}$$

$$\chi(np)=1{\cdot}56 \text{ MeV}$$

—and we may set $0{\cdot}05$ MeV as a generous estimate of probable error for any of these values. It is to be noted that there appears to be a slight binding effect between an unpaired neutron and an unpaired proton in different quantum states in the same nucleus, and, at least as a suggestive numerical result, that the ratio $\chi(p)/\chi(n)$, given by the above figures as $1{\cdot}40\pm0{\cdot}12$, is the same as the ratio of the absolute values of the magnetic moments of neutron and proton (1·46)—to well within the limits of uncertainty of the data. In this connection we may remark that the χ terms have, in fact, sometimes been referred to as spin-dependent terms in the literature (see Saha and Saha 1946).

Our method of combining experimental results up to this point in the discussion has been such as to bring into prominence the effect of the nucleon-parity dependent terms in the mass formula. We can suppress these terms entirely, so long as they remain constant over finite ranges of N and Z, and so examine shell closure (or other) effects, by a slight change in procedure. It has just been stated that the 'theoretical' expression for $B_n\binom{A}{Z}-B_n\binom{A-1}{Z-1}$ involves the quantity $\chi(np)-\chi(n)-\chi(p)$. If we write $\chi'(np)$ for this quantity, the full statement is that, whatever the parity of Z, $B_n\binom{A}{Z}-B_n\binom{A-1}{Z-1}$ is given 'theoretically' by an expression containing $\chi'(np)$ when $A-2Z$ is odd and $-\chi'(np)$ when $A-2Z$ is even. Thus if we define (Feather 1953)

$$\begin{aligned} {}_D\Delta_p(Z) &= B_p\binom{2Z+D}{Z}-B_p\binom{2Z+D-1}{Z} \\ &= B_n\binom{2Z+D}{Z}-B_n\binom{2Z+D-1}{Z-1} \end{aligned}$$

and

$${}_D\Delta'_p(Z)={}_{(D-2)}\Delta_p\,(Z+1)-{}_D\Delta_p(Z)$$

then, clearly, the 'theoretical' expression for ${}_D\Delta'_p(Z)$ does not contain any nucleon-parity dependent term, whatever the parity of Z or D.

To be precise, we have, from (11)

$$
{}_D\Delta'_p(Z)=\frac{8\beta(D-1)}{A(A-1)(A-2)}
+2\delta\left[Z\left(\frac{1}{(A-1)^{1/3}}-\frac{1}{A^{1/3}}\right)-(Z-1)\left(\frac{1}{(A-2)^{1/3}}-\frac{1}{(A-1)^{1/3}}\right)\right], \quad \ldots\ldots \quad (17)
$$

or, sufficiently accurately, when $Z\gg 1$,

$$
{}_D\Delta'_p(Z)=\frac{8\beta(D-1)}{A^3}+\frac{2\delta}{3A^{4/3}}\left(1-\frac{4Z}{3A}\right). \quad \ldots \quad (17\,a)
$$

Inserting numerical values in (17) and (17 *a*), as appropriate, we obtain the 'sample' values of ${}_D\Delta'_p(Z)$ (calculated) given below:

Z	7	16	90
D	2	3	51
${}_D\Delta'_p(Z)$ (Mev)	0·052	0·0096	0·0008

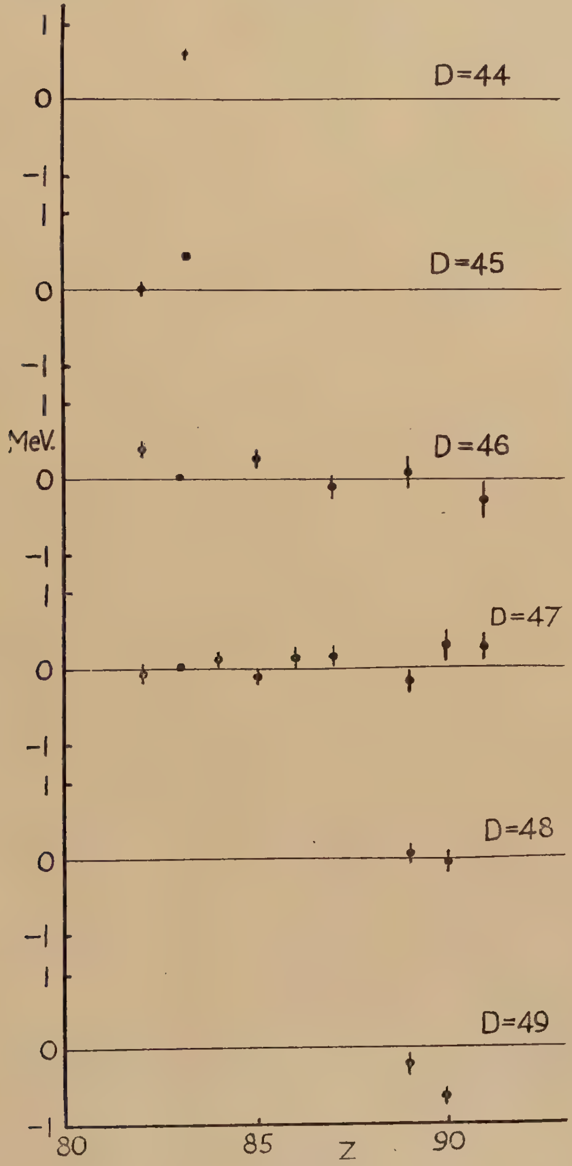
These sample values are sufficient to show (Feather 1953) that, so far as the v. Weizsäcker mass formula (with nucleon-parity dependent terms included) is concerned, our new parameter ${}_D\Delta'_p(Z)$ should not differ appreciably from zero except for $Z<10$, and then by amounts which will not always be outside the limits of experimental error. Conversely, experimental values of ${}_D\Delta'_p(Z)$ significantly different from zero (with small corrections when $Z<10$) must be regarded as demonstrating the failure of the mass formula in particular cases—either because of its omission of shell-closure or other effects, or because of the non-constancy of the terms χ now included.

Figure 3 gives experimental values of ${}_D\Delta'_p(Z)$, so far as these are available, for $44\leqslant D\leqslant 49$. In all, 22 points are plotted on this figure—and not more than about 5 of them lie significantly off the line $\Delta'=0$. To a considerable extent, then, predictions are verified over this range of D—and, before it becomes profitable to discuss the apparent discrepancies, further experimental information is obviously required, however difficult it may be to obtain.

Turning to the range of the lighter nuclei, the behaviour of the parameter Δ' shows features of greater interest. Experimental values for $D=1$ and $D=2$ are plotted in fig. 4 and those for $3\leqslant D\leqslant 8$ in fig. 5. The contrast in behaviour represented by the two 'curves' of fig. 4 is particularly striking. Clearly the mass formula is quite inadequate for accurate calculation of ${}_2\Delta'_p(Z)$, but, surprisingly, the inadequacies of the formula appear to cancel out completely in the calculation of ${}_1\Delta'_p(Z)$. The explanation of this last effect is quite simple, though not directly along the lines implied by the form of words used in its description. We have, in general,

$$ {}_D\Delta'_p(Z) = B_p\binom{2Z+D}{Z+1} - B_p\binom{2Z+D-1}{Z+1} - \left[B_p\binom{2Z+D}{Z} - B_p\binom{2Z+D-1}{Z}\right]. $$

$_D\Delta'_p(Z)$ Fig. 3

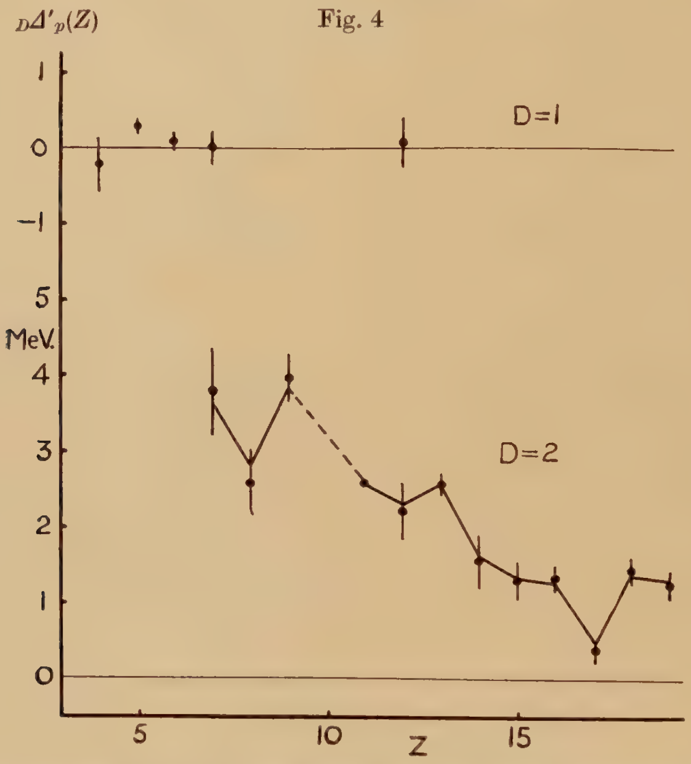


Replacing the first two terms of this expression by the corresponding difference of neutron binding energies, and re-arranging, we obtain

$${}_D\Delta'_p(Z)=B_n\binom{2Z+D}{Z+1}-B_p\binom{2Z+D}{Z}-\left[B_n\binom{2Z+D-1}{Z}-B_p\binom{2Z+D-1}{Z}\right]$$

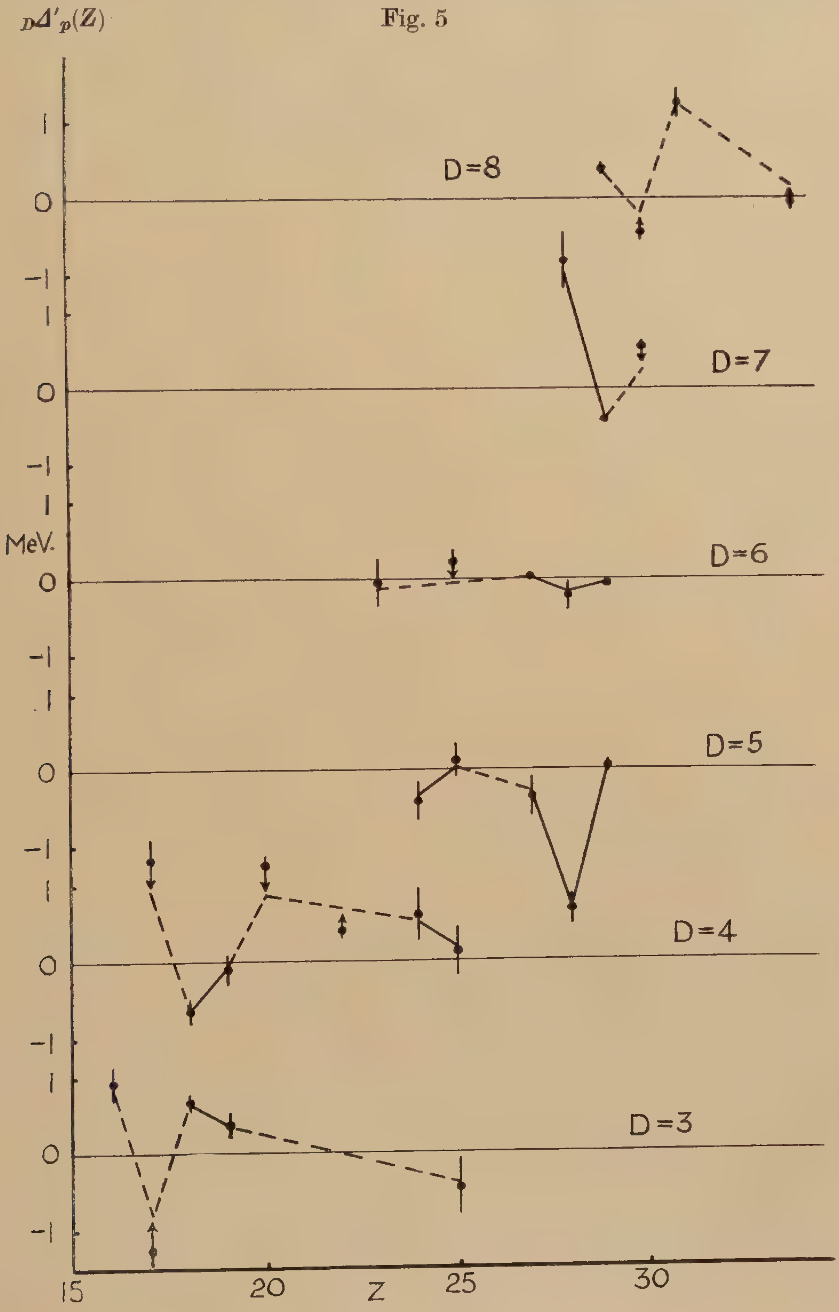
—and we now note (taking the terms in order) that the neutron excess of the species $\binom{2Z+D}{Z+1}$ is $D-2$, that the proton excess of $\binom{2Z+D}{Z}$ is $-D$, and that the species $\binom{2Z+D-1}{Z}$ is of neutron excess $D-1$ (and therefore of proton excess $1-D$). When $D=1$ (and only for this value of D) this statement takes on special symmetry. In this case the neutron excess of $\binom{2Z+D}{Z+1}$ and the proton excess of the neighbouring isobar $\binom{2Z+D}{Z}$ are equal (at -1) and the species $\binom{2Z+D-1}{Z}$ is of neutron excess (and also proton excess) zero. It is then easy to see that if we assume the complete symmetry-equivalence of neutrons and protons ${}_1\Delta'_p(Z)$ can differ from zero only because of charge dependent effects—whatever the precise nature of the binding forces concerned. This is a more general statement than is formally provided by (17)—though this equation, too,

Fig. 4



shows that $D=1$ constitutes a special case. In this case the first ('symmetry') term of (17) vanishes identically, and only the second ('electrostatic') term remains. For $Z=4$ the numerical value of this term is 0·0104 MEV, and for $Z=12$ it is 0·0021 MEV. It is clear, then, that the validity of the v. Weizsäcker formula is not in question in explaining

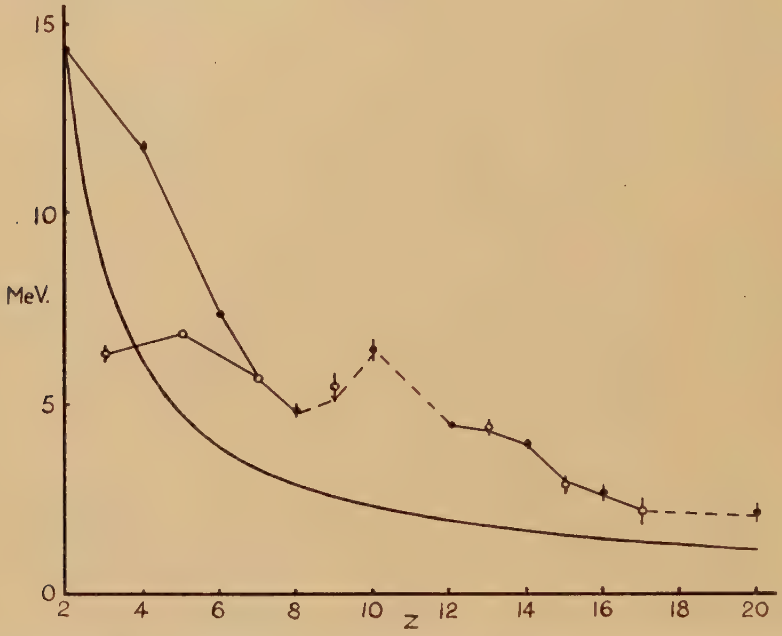
Fig. 5



the results for $D=1$ exhibited on fig. 4, except in so far as the term in δ is concerned—and that in this particular there is no obvious discrepancy.

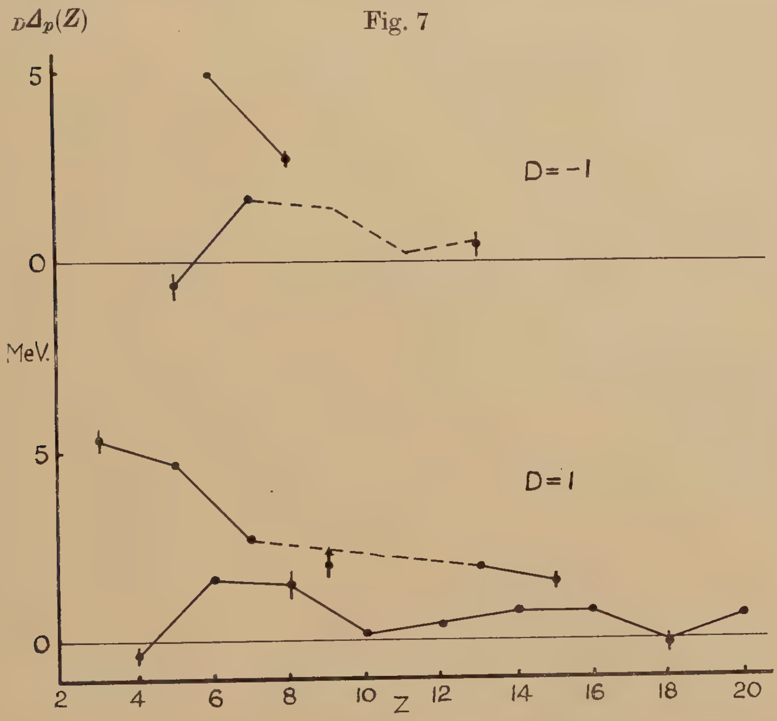
Having established the fundamental basis underlying the behaviour of ${}_1\Delta'_p(Z)$, we may note that one point on fig. 4, that for $Z=5$, appears to lie significantly off the straight line, ${}_1\Delta'_p=0$. In view of our discussion this raises the suspicion that in this case the experimental results are at fault. The results directly employed in the calculation of ${}_1\Delta'_p(5)$ were the reaction energies of the (p, n) reactions with ${}^{11}_{5}$B and ${}^{9}_{4}$Be, the energies of negative β-disintegration of ${}^{10}_{4}$Be and the neutron, and the total energy of positron disintegration of ${}^{10}_{6}$C, and suspicion naturally fastens on the last of these. It would appear likely that, if the main disintegration mode of ${}^{10}_{6}$C leads to the formation of ${}^{10}_{5}$B in the 0·71 MEV excited state, then the maximum energy of the positrons is 1·9 MEV rather than 2·2±0·1 MEV as given in the literature. Determination of the (p, n) reaction energy with ${}^{10}_{5}$B would clinch the matter (see Sherr and Gerhart 1952).

Fig. 6



Before returning to the discussion of figs. 4 and 5 it may be profitable to consider first the trend of ${}_0\Delta_p(Z)$ with Z as represented by the experimental values plotted in fig. 6. For $D=0$ we are not able to deduce experimental values of ${}_D\Delta'_p(Z)$, just because information is not available regarding ${}_{-2}\Delta_p(Z+1)$. For this reason some of the information displayed in fig. 6 would be left out of consideration altogether if it were not presented in this way—uncompensated as it is for nucleon parity dependent effects. As already stated, the result of this lack of compensation is that the observed value of ${}_0\Delta_p(Z)$ should in any case be greater than the value calculated on the basis of (11) by an amount $\chi(n)+\chi(p)-\chi(np)$, whether Z is odd or even. In fig. 6 these uncorrected ' calculated ' values of ${}_0\Delta_p(Z)$ are given by the smooth curve. A very striking feature of the experimental results is at once evident from the figure. In line with the above considerations it appears that parity with respect to Z is of no great consequence in determining the lie of the points so long as $Z>7$, but for $Z<7$ a very large parity-dependent effect shows up. The effect of an additional (odd-numbered) neutron in increasing the energy of binding of the corresponding odd-numbered proton is less by some 2·7 MeV when $Z=5$, and by some 6·7 MeV when $Z=3$, than might have been expected from the behaviour of ' neighbouring ' additional even-numbered neutrons in increasing the binding energies of corresponding even-numbered protons. Moreover, from the general aspect of fig. 6, we conclude that the points for odd $Z (Z<7)$, rather than the points for even Z in the same range, are the anomalously placed points in the figure. A similar effect shows up over the same range of Z with ${}_{-1}\Delta_p(Z)$ and ${}_1\Delta_p(Z)$, as exhibited in fig. 7—though all variations of any significance disappear when the difference ${}_{-1}\Delta_p(Z+1)-{}_1\Delta_p(Z)(={}_1\Delta'_p(Z))$ is taken (see fig. 4).

Fig. 7



Comparing figs. 6 and 7 it appears that the nucleon-parity dependent effect persists beyond $Z=7$ in considerably greater prominence with ${}_1\Delta_p(Z)$ than with ${}_0\Delta_p(Z)$, and it is evident that when $Z\leqslant 7$ the upper branch of the curve showing the trend of Δ is defined by the points for even Z so far as ${}_{-1}\Delta_p(Z)$ and ${}_0\Delta_p(Z)$ are concerned, and by the points for odd Z when ${}_1\Delta_p(Z)$ is in question. This is not the usual alternation in behaviour as the parity of the isotopic number D alternates; for its formal expression it is necessary to say that the 'high' values of ${}_D\Delta_p(Z)$, at least when $Z \leqslant 7$, are those for which $2Z+D=4n$ or $2Z+D=4n-1$, and the 'low' values those for which $2Z+D=4n-2$ or $2Z+D=4n-3$, n being integral throughout. From the evidence of fig. 7 this form of statement remains valid for $D=1$ beyond $Z=7$, almost certainly as far as $Z=20$. The implication of the statement is that, whether neutrons or protons are concerned, the binding of a given nucleon is increased, by the addition of a nucleon of opposite type to a nucleus, by a 'large' amount when that addition results in a final nucleus of mass number $4n$ or $4n-1$, and by a 'small' amount when the addition results in a final nucleus of mass number $4n-2$ or $4n-3$. If this result represents one aspect of the 'α-unit structure' of the lightest nuclei, the evidence from figs. 6 and 7 seems to show that this structural pattern becomes largely obliterated by patterns of greater complexity when more than 4 α-units have been assembled in a single nucleus. Beyond this stage, although the distinction between final nuclei of mass numbers $4n-1$ and $4n-3$ appears to persist in the sense implied in our previous statement, the corresponding distinction between final nuclei of mass numbers $4n$ and $4n+2$ has largely disappeared. Figure 6 provides evidence for an unusual degree of stability for the nucleus ${}^{20}_{10}\mathrm{Ne}$ (conceivably a tetrahedral arrangement of α-units with an α-core), but otherwise no further feature of obvious significance emerges.

Returning to figs. 4 and 5 we now take the results for the various series $D=$constant in turn. For $D=2$ (fig. 4) we may say broadly that the value of Δ' is significantly high for $Z=9$ and $Z=13$, and significantly low for $Z=17$. Apart from these values, this quantity, which on the basis of (11), with constant nucleon parity dependent terms included, should be effectively zero throughout the range, decreases monotonically from about 3·5 MeV at $Z=7$ to 1·2 MeV at $Z=19$. So far as this monotonic decrease is concerned, remembering that ${}_2\Delta'_p(Z)={}_0\Delta_p(Z+1)-{}_2\Delta_p(Z)$, and referring to fig. 6 for ${}_0\Delta$, we conclude that the variations of ${}_2\Delta$ over the range $7 \leqslant Z \leqslant 19$ are fairly closely parallel to those of ${}_0\Delta$ over the range $8 \leqslant Z \leqslant 20$, but that absolute values of ${}_2\Delta$ are somewhat less than the 'calculated' values, whereas absolute values of ${}_0\Delta$ are considerably greater than the 'calculated' values over the range in question. This last difference probably derives from the fact that whereas ${}_0\Delta$ expresses the increase in the binding energy of a given nucleon due to the addition of a nucleon of opposite type belonging to the same α-unit, ${}_2\Delta$ measures the increase in proton (neutron) binding energy involved when the added neutron (proton) belongs to the 'next' ('last') α-unit to form in the nucleus. The monotonic decrease exhibited in the curve for $D=2$ on fig. 4 then appears as representing the slowly diminishing persistence of α-unit structure in nuclei having $8 \leqslant Z \leqslant 20$—and the anomalous positions of the points for $Z=9$ and 13 as expressing the unusual degree of stability of the nuclei ${}^{20}_{10}\mathrm{Ne}$ (already referred to) and ${}^{28}_{14}\mathrm{Si}$ (this case less marked than the other). The position of the point for $Z=17$ is less easy to explain. This point, like many others on figs. 3, 4 and 5, has been plotted, without knowledge of the four proton binding energies individually—or the two differences Δ individually—which are involved in the definition of Δ', by direct use of the formula (Feather 1953)

$$_D\Delta'_p(Z)=T_\beta^-\binom{2Z+D}{Z}-T_\beta^-\binom{2Z+D-1}{Z}-T_\beta^-\binom{2Z+D-1}{Z-1}+T_\beta^-\binom{2Z+D-2}{Z-1}. \quad . \quad . \quad . \quad (18)$$

Clearly, points which are plotted by use of (18), individual binding energies being unknown, lack full 'documentation', and if they prove to be anomalously placed may pose problems in interpretation, the detailed answers to which are at present obscure. This is the case with the point for $Z=17$, $D=2$ (fig. 4).

In fig. 5 the points corresponding to the (Z, D) values (17, 3), (18, 4), (19, 4) and (28, 5) appear significantly low and those corresponding to (28, 7) and (31, 8) significantly high by comparison with neighbouring points in each case. Again, most of these anomalously-placed points are incompletely documented in the sense just defined; only for $_4\Delta'_p(19)$ and $_5\Delta'_p(28)$ are all four last-proton (and corresponding last-neutron) binding energies individually known. Treating these two fully-documented anomalies in terms of last-neutron binding energies (because these, rather than last-proton binding energies are more often the primary data of experiment), we may say that $_4\Delta'_p(19)$ is low because the addition of the 20th proton causes a smaller than 'normal' increase in binding of the 22nd neutron (rather than because the addition of the 19th proton causes a larger than normal increase in binding of the 23rd neutron)—and that $_5\Delta'_p(28)$ is particularly low apparently as a result of the combined effects of a high value of $B_n\binom{61}{28}$ and a low value of $B_n\binom{61}{29}$, $B_n\binom{60}{28}$ also being high. The former anomaly cannot be explained away on the basis of experimental uncertainties, the value of $_4\Delta'_p(19)$ being given most directly in terms of $T_\beta^-\binom{40}{18}$, $T_\beta^-\binom{41}{18}$ and $T_\beta^-\binom{42}{19}$—all known with reasonable certainty—and the energy of the (p, n) reaction with $^{41}_{19}$K. Adopting the cautious view that the last-mentioned quantity is known only as a lower limiting value, we recognize our crude experimental value for $_4\Delta'_p(19)$ as an upper limit. This particular anomaly, then, cannot be less serious than it appears from fig. 5. Reviewing the values of the binding energies of the 22nd neutron in other nuclei, the anomaly appears in fact to persist throughout: for even N there appears to be a significant change in the average slope of the curves of $B_n\binom{A}{Z}$, N constant, against Z, between $N=20$ and $N=22$. For $N=20$ (and very similar slopes occur for $N=18$ and $N=16$, also) $B_n\binom{A}{Z}$ increases by 5·3 MeV as Z increases from 17 to

20, whereas for $Z=22$ the corresponding increase is only 3·0 MeV. This may justifiably be regarded as a neutron-shell-closure effect at $N=20$.

In respect of ${}_5\Delta'_p(28)$ the primary data are $T_\beta^-\binom{61}{28}$, $T_\beta^-\binom{60}{28}$, $T_\beta^-\binom{60}{27}$ and the energy of the (p, n) reaction with ${}^{59}_{27}$Co. Not more than about 1·0 MeV towards a discrepancy of at least 1·6 MeV could here be gained by assuming that this (p, n) reaction energy is in fact greater than experimentally determined—but the discrepancy in Δ' would effectively be removed completely if it were shown that ${}^{61}_{29}$Cu emits two γ-rays each of about 1·5 MeV energy in cascade, rather than a single radiation. Intensity measurements do not appear to be available, thus the question remains open. The value of $B_n\binom{61}{29}$ would no longer be low if the latter suggestion were substantiated; on the other hand $B_n\binom{61}{28}$, being independently known on the basis of the energy of the (n, γ) reaction with ${}^{60}_{28}$Ni, would still be high. This aspect of the anomaly would not be removed—and the assumption that the ground-to-ground transition was unobserved in this determination would merely serve to aggravate it. It appears in the upshot that the anomaly is largely 'real'. With the 'incompletely documented' ${}_7\Delta'_p(28)$ the assumption of a cascade of two similar γ-rays (of about 1·3 MeV energy) with ${}^{62}_{27}$Co offers the possibility of almost entirely removing the numerical discrepancy recorded in the figure, but this suggestion, like the former one, is somewhat artificial, with no positive evidence in its favour. After all, 28 has been regarded as a 'magic' number by some authors—and the anomalies at present in question certainly involve this nucleon number in one way or another.

Experimental results for $T_\beta^-\binom{40}{18}$, $T_\beta^-\binom{38}{17}$, $T_\beta^-\binom{39}{18}$ and $T_\beta^-\binom{39}{17}$ allow of very little latitude in respect of the value of ${}_4\Delta'_p(18)$, thus the discrepancy represented on fig. 5 must be taken as definite in this case, but for ${}_3\Delta'_p(17)$ the position is less well established. A little would be gained towards removing the discrepancy if it were shown that the full reaction energy was not measured for the (p, n) reaction with ${}^{37}_{17}$Cl, and it might prove necessary to allow slightly more than the 0·5 MeV allowed in the figure for the energy of capture transformation of ${}^{36}_{17}$Cl at present unknown, but it is very unlikely that the full discrepancy of about 1·6 MeV could be removed by these adjustments. Again, both these discrepancies could reflect shell-closure effects around $N=20$ or $Z=20$, as the discrepancy with ${}_4\Delta'_p(19)$ has been shown to do; for that reason alone they are the less surprising. Only it should be stated, if they are to be explained in this way, that the general trend of the curves of $B_n\binom{A}{Z}$, N constant, against Z, for N odd, is somewhat more confused than is the case for the corresponding curves for N even. For that family of curves we have already remarked on a fairly clear-cut break between $N=20$ and $N=22$; for N odd the curve for $N=19$ has more the nature of a transition curve between the pairs of curves for $N=15$, 17 and $N=21$, 23.

For the point (31, 8) in fig. 5 direct determination of the disintegration energies $T_{\beta}^{-}\binom{70}{31}$, $T_{\beta}^{-}\binom{68}{30}$, $T_{\beta}^{-}\binom{69}{31}$ and $T_{\beta}^{-}\binom{69}{30}$ fixes the final value of Δ' without much ambiguity, and further consideration shows that this incompletely documented anomaly has its most probable origin in the value of $B_n\binom{69}{31}$ which is surprisingly low. Determined independently by the (γ, n) reaction with $^{69}_{31}$Ga, the experimental binding energy in this case is an upper limiting value: no uncertainty regarding interpretation, therefore, can 'improve' the position. In fact a wider survey indicates that the pair of values **36, 38** has much in common with the pairs 28, 30 and 20, 22 for the diagram in which $B_n\binom{A}{Z}$, N constant, is plotted against Z. These are the pairs of values of N even $(16 \leqslant N \leqslant 46)$ between which the major decreases in mean slope of the curves for N constant occur. On this basis there would appear to be as much justification for regarding 36 as a 'magic' number as there is for regarding 28 or 20 similarly, at least in respect of neutron shell closure.

For $8 < D < 44$ values of ${}_D\Delta'_p(Z)$ are not plotted in this paper, experimental information being insufficient—or of insufficient accuracy—for trends to be exhibited satisfactorily by graphical presentation. For completeness, however, individual values (in MeV) are collected in tabular form below.

Z	35	34	40	42	34	38	39	40	41	42
D	10	11	11	11	12	12	12	12	12	12
Δ'	0·10 ±0·20	0·34 ±0·28	−2·28 ±0·75	−1·20 ±0·55	−0·04 ±0·40	−0·49 ±0·20	−0·13 ±0·04	0·59 ±0·60	0·93 ±0·50	1·07 ±0·60

Z	33	36	38	39	40	44	39	44	49	41
D	13	13	13	13	13	13	14	14	14	15
Δ'	−0·44 ±0·50	−0·48 ±0·03	0·30 ±0·03	0·28 ±0·03	−0·17 ±0·28	−0·21 ±0·50	−0·72 ±0·22	0·85 ±0·44	0·86 ±0·30	1·69 ±0·24

Z	46	48	49	57	59
D	15	16	18	27	27
Δ'	0·08 ±0·30	0·63 ±0·21	−0·75 ±0·32	−0·62 ±0·12	1·22 ±0·30

This table includes all 'known' values of Δ' not already given in figs. 3, 4 and 5. There are a great many other cases, of course, in which three of the four terms necessary for the calculation of Δ' are known, but not so the fourth. In these cases, quite frequently, a limiting value of Δ' may be deduced which is significant for discussion—or a rough quantitative prediction may be made concerning an unknown energy of β-disintegration which is of interest on its own account—but to include these tentative considerations here would be to extend the paper unduly. We confine ourselves, therefore, to certain comments on those values of Δ' included in the table which differ from zero by amounts of the order of 1 MeV or more. The (Z, D) specifications involved are (40, 11), (42, 11), (41, 12), (42, 12), (41, 15) and (59, 27), respectively.

To a large extent the anomalies in the first four values of Δ' specified in this list can be attributed to irregularities in last-neutron binding energies characteristic of the various isotopes of niobium ($Z=41$). In an overall survey it has to be concluded that present experimental values of $B_n\binom{91}{41}$ and $B_n\binom{94}{41}$ are low, and that of $B_n\binom{95}{41}$ high, when compared with the values of $B_n\binom{92}{41}$ and $B_n\binom{93}{41}$ which appear more or less normal in relation to corresponding values for neighbouring species. The anomalous value of ${}_{15}\Delta'_p(41)$, also, though 'incompletely documented', almost certainly entails a low value for $B_n\binom{96}{41}$—and probably a high value for $B_n\binom{96}{40}$. It is very curious indeed that these five anomalies should be concentrated effectively in a single element (anomalous values of $B_n\binom{A}{Z}$ characterized by the single proton number $Z=41$): it is possible that for $Z\sim 41$ cases of unsuspected isomerism remain to invalidate conclusions drawn from experimentally determined reaction energies, but that all these hypothetical errors of interpretation should at this stage issue in incorrect last-neutron binding energies for niobium isotopes would appear to leave something still to be explained, even if the present calculations are shown to involve such errors. The greatest anomaly appears to be with $B_n\binom{91}{41}$—which is some 2·8 MeV low with respect to the average of $B_n\binom{90}{40}$ and $B_n\binom{92}{42}$—but this binding energy is known only by combining ten independent experimental determinations as follows:

$$B_n\binom{91}{41}=B_n\binom{91}{40}+2T_\beta^-\binom{1}{0}-T_\beta^-\binom{90}{40}-T_\beta^-\binom{91}{41}+B_n\binom{92}{40}-B_n\binom{93}{42}$$
$$+B_n\binom{93}{41}-B_n\binom{92}{42}+Q_7\binom{92}{40}+Q_7\binom{93}{41}.$$

Here $B_n\binom{91}{40}$ is known without uncertainty as a result of the determination of (d, p) and (γ, n) reaction energies, $B_n\binom{92}{40}$ and $B_n\binom{93}{42}$ depend on (d, p) reaction energies alone, $B_n\binom{93}{41}$ and $B_n\binom{92}{42}$ on (γ, n) reaction energies alone, and $Q_7\binom{A}{Z}$ in this connection denotes the reaction energy of the (p, n) reaction with $\binom{A}{Z}$ as target nucleus. There is evidently considerable possibility of interpretative error in the determination—and 'statistically' the balance of probability is that the final value will be an underestimate rather than the reverse—but only further experimental investigation can show whether the accepted value is in error or not. Compared with the complexity of this case, the situation in respect of $B_n\binom{94}{41}$ is entirely simple: if there is any error in the currently accepted (low) value of this binding energy it arises from the non-observance of the ground-to-ground state transition in the (d, p) reaction with ${}^{93}_{41}\mathrm{Nb}$.

In respect of the high value of ${}_{27}\Delta'_p(59)$ no precise explanatory suggestions can be made in the present state of knowledge; it is possibly worthy of note, however, that besides this value of Δ', only one other is known at present which involves neutron numbers in the range $82 \leqslant N \leqslant 88$ in which effects consequent upon shell closure at $N = 82$ are likely to show up.

APPENDIX I

The table of last-nucleon binding energies given in this appendix is constructed using proton number, Z, and isotopic number, $D(=A-2Z)$, as specifying constants. Each space in the table then refers to a particular nucleus $\binom{A}{Z}$, and the energy values inserted in a particular space give (in order downwards) the last-neutron binding energy, B_n, the probable error in this quantity, δ_n, the last-proton binding energy, B_p, and the corresponding probable error, δ_p, for the nucleus in question, all values being in MEV. This arrangement is shown symbolically in the otherwise vacant space for $\binom{30}{12}$, on the first page of the table.

A value in *italics* is either a 'primary' value, deriving directly from an induced reaction energy, or a value based on more than one experimentally determined quantity and such that the last step in its derivation involves the use of an induced reaction energy.

A value in smooth brackets, ( ), is either a primary value assigned without full justification to a particular isotope, or a 'secondary' value based on a primary value so assigned.

A value in square brackets, [ ], has been derived by use of 'exact' nuclear masses.

No information is available concerning last-nucleon binding energies for any nuclei having $61 \leqslant Z \leqslant 72$: our table therefore omits this range of Z entirely. As it is presented the table also fails to include four binding energies concerning which information is available. These four are given here for sake of completeness: $B_p\binom{24}{13} = \mathit{1{\cdot}8 \pm 0{\cdot}3}$, $B_n\binom{150}{60} = \mathit{7{\cdot}4 \pm 0{\cdot}2}$, $B_n\binom{204}{80} > 7{\cdot}0 \pm 0{\cdot}2$, $B_n\binom{205}{80} < 5{\cdot}9 \pm 0{\cdot}2$.

Z \ D	−2	−1	0	1	2	3	4	5	6
12			*16·59*	*7·323*	*11·106*	*6·433*			B_n
			0·02	0·008	0·010	0·007			δ_n
		7·37	11·71	12·07	14·0				B_p
		0·05	0·01	0·01	0·3				δ_p
11				12·25	*6·957*	9·2			
				0·05	0·009	0·3			
		2·45		8·62	10·39				
		0·03		0·05	0·05				
10			*16·73*	*6·755*		*5·190*			
			0·07	0·008		0·008			
		6·36	12·69	12·85					
		0·30	0·07	0·07					
9			⩽9·52	*10·40*	*6·599*				
			0·20	0·30	0·008				
		0·62	⩽*6·00*	*7·95*	10·3				
		0·03	0·35	0·30	0·3				
8		13·25	*15·6*	*4·143*	⩽8·45	4·25			
		0·20	0·2	0·006	0·35	0·45			
	4·65	7·35	12·1	13·55	⩽*16·35*				
	0·20	0·02	0·2	0·30	0·20				
7		*20·45*	*10·55*	*10·834*	2·70	5·65			
		0·10	0·01	0·008	0·35	0·45			
	0·30	*1·95*	7·547	10·207	*11·2*				
	0·10	0·01	0·009	0·008	0·6				
6		13·38	*18·77*	*4·950*	*8·174*	*1·7*			
		0·10	0·06	0·007	0·009	0·5			
	3·75	*8·70*	16·01	17·60					
	0·10	0·01	0·06	0·06					
5		18·35	*8·44*	*11·461*	*3·362*				
		0·30	0·02	0·011	0·006				
	0·40	−*0·185*	6·59	11·24					
	0·35	0·003	0·02	0·02					
4			*18·94*	*1·667*	*6·808*				
			0·15	0·002	0·006				
		5·599	*17·29*	*16·93*					
		0·008	0·15	0·15					
3			*5·35*	*7·245*	*2·034*				
			0·20	0·008	0·003				
		−*1·6*	*4·7*	*10·03*					
		0·2	0·3	0·04					
2			[20·58]	−*0·94*	*1·9*				
			0·02	0·05	0·3				
		5·493	[19·82]						
		0·015	0·02						
1			*2·226*	*6·256*					
			0·003	0·007					
			2·226						
			0·003						

Z \ D	−1	0	1	2	3	4	5	6	7
24			<7·2 0·2	*13·4* 0·2 8·4 0·3	[9·22] 0·25 *9·62* 0·20	*11·80* 0·25 *10·27* 0·25	*7·75* 0·20 10·77 0·35	*10·33* 0·35	5·77 0·25
23				6·22 0·40	<12·20 0·06 <7·37 0·05	[8·00] 0·35 <7·2 0·4	*11·15* 0·20 [7·98] 0·25	*7·25* 0·05 8·65 0·15	
22				*13·3* 0·2 10·5 0·2	*8·74* 0·10 10·34 0·10	*11·05* 0·40 10·88 0·40	*8·15* 0·05 11·25 0·15	[<10·35] 0·40 <11·55 0·40	6·6 0·3
21	1·65 0·15					8·9 0·2 8·4 0·2	10·51 0·10	7·8 0·4	10·05 0·20
20	6·3 0·3	15·28 0·20 8·35 0·05	*8·40* 0·05 8·98 0·06	*11·37* 0·12 10·15 0·12					
19	2·04 0·25		*13·2* 0·2 6·70 0·25	*7·77* 0·03 7·55 0·03	*10·20* 0·06 7·85 0·15	*7·35* 0·10 9·12 0·15			
18		14·7 0·2 8·50 0·03	8·41 0·20 *8·35* 0·20		6·92 0·25 10·95 0·25	9·90 0·15 12·44 0·15	6·08 0·20		
17	2·42 0·13	10·75 0·35 4·5 0·3		*8·56* 0·03 7·95 0·03	*9·95* 0·20		8·41 0·30		
16	6·25 0·25	14·71 0·12 8·85 0·01	*8·65* 0·01 9·57 0·01	*10·85* 0·20 10·33 0·20					
15	3·02 0·15	11·25 0·25 5·8 0·2	12·1 0·2 7·30 0·02	*7·93* 0·01 8·64 0·02	10·09 0·02	6·0 0·3			
14	7·71 0·12	16·84 0·11 11·56 0·03	*8·47* 0·01 12·31 0·03	*10·61* 0·01 13·58 0·25	*6·59* 0·01				
13	*17·0* 0·4 *2·2* 0·2	*11·5* 0·2 6·41 0·06	12·99 0·06 8·29 0·01	*7·72* 0·01 9·58 0·02					

Z ╲ D	0	1	2	3	4	5	6	7	8	9	10
31					5·21 0·20	< 11·8 0·2 < 5·8 0·2	> 7·65 0·25 6·45 0·25	*10·1* 0·2 6·40 0·25		*9·05* 0·20	
30				9·20 0·15 6·69 0·15	11·85 0·06 7·69 0·05	*7·92* 0·05 7·71 0·05	*11·15* 0·20 8·81 0·20	*7·0* 0·2 8·85 0·20	*10·15* 0·20 9·94 0·20	6·32 0·25	*9·2* 0·2
29			6·35 0·25	10·64 0·20 5·54 0·03		*10·85* 0·15 6·13 0·15	*7·90* 0·05 7·18 0·05	10·05 0·05 7·59 0·05	6·96 0·20 8·28 0·20	10·30 0·35	
28			*11·7* 0·2 7·7 0·2	*9·01* 0·10 *8·39* 0·20	*11·45* 0·10 9·6 0·1	*8·55* 0·03 10·58 0·03		6·85 0·15 9·65 0·40	9·64 0·05	6·27 0·08	
27		5·10 0·06		5·46 0·10	*8·3* 0·3 6·1 0·3	*10·25* 0·20 7·17 0·20	*7·6* 0·1 8·4 0·1	10·1 0·2			
26	7·75 0·40	10·05 0·35 7·3 0·3	*13·8* 0·2 9·2 0·3	*9·34* 0·05 9·0 0·2	11·16 0·04 10·17 0·04	*7·65* 0·10 10·58 0·10	9·2 0·3	6·39 0·25			
25		5·0 0·4	10·5 0·4 6·28 0·25	11·89 0·20 *6·37* 0·20	9·55 0·35 *8·17* 0·35	*10·0* 0·2 *7·84* 0·20	*7·24* 0·03 9·31 0·15				

Z \ D	5	6	7	8	9	10	11	12	13	14	15	16
38						*9·5* 0·2	*8·5* 0·2 9·5 0·2	*11·15* 0·20 10·64 0·20	*7·05* 0·10 11·40 0·10	8·25 0·20 ⩾11·95 0·55	5·20 0·25 ⩾10·1 0·4	
37									10·0 0·2	6·29 0·20 9·14 0·20	⩽7·7 0·5 ⩽9·30 0·55	
36	(⩽7·0) 0·3		7·95 0·20	(11·14) 0·20 (8·50) 0·20								7·54 0·35 14·7 0·6
35			(10·53) 0·20 (5·16) 0·20		*10·6* 0·2 6·2 0·3	(7·31) 0·20 (6·68) 0·20	*9·95* 0·20 7·28 0·20					
34					*(7·3)* 0·2		6·8 0·3	*(9·35)* 0·20	6·66 0·25			
33					*10·1* 0·2 6·75 0·20							
32	6·0 0·2						6·25 0·25					

Z \ D	7	8	9	10	11	12	13	14	15	16	17
45			(4·51) 0·20	(7·94) 0·30 (*5·4*) 0·3			*9·35* 0·20 6·45 0·25	*6·81* 0·20 6·78 0·20	(9·06) 0·25		
44					(*7·05*) 0·20	(*9·5*) 0·2 (9·0) 0·2			6·48 0·25		
43	>10·5 0·5 4·39 0·20	8·75 0·45 4·83 0·45	(9·43) 0·40 (*4·35*) 0·40					(7·10) 0·35 (7·68) 0·35			
42	4·47 0·55	*13·28* 0·15 7·80 0·35	*8·31* 0·20 *7·45* 0·30	*9·91* 0·45 8·66 0·40	(*7·95*) 0·25 (9·35) 0·50		*6·9* 0·2 9·28 0·30				
41		7·1 0·4	9·97 0·65 *4·77* 0·55	*8·66* 0·50 *6·23* 0·25	*8·7* 0·2 *6·2* 0·3	*7·26* 0·10 6·54 0·10	(9·20) 0·50		8·13 0·30 *5·53* 0·35		
40	⩾6·11 0·35	<12·3 0·3 <8·0 0·3	>9·8 0·1 8·60 0·10	*12·3* 0·2 8·7 0·2	*7·2* 0·1 8·62 0·10	*8·73* 0·10 9·51 0·10	6·92 0·30 9·62 0·35				3·65 0·35
39		⩽5·1 0·3	⩾10·40 0·35 *6·0* 0·3	*9·17* 0·30 6·67 0·20	12·21 0·10 7·73 0·10	7·3 0·2 8·0 0·2	7·84 0·10 7·59 0·10	6·80 0·25 9·2 0·3	7·3 0·4		

Z \ D	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
52								<7·94 0·30	>9·25 0·20 7·89 0·20			*(6·5)* 0·2			
51					⩽5·7 0·4					9·3 0·3 5·8 0·3	*6·64* 0·20 6·24 0·20				
50						(7·68) 0·20 (8·90) 0·20	8·94 0·35 8·79 0·35		*9·25* 0·15 10·35 0·20	*6·60* 0·20 ⩾7·3 0·3		*6·2* 0·3			*8·50* 0·15
49	5·83 0·25					<9·9 0·2 <6·36 0·20		*(9·05)* 0·20 *(6·75)* 0·30	*6·59* 0·20 7·26 0·20		⩽9·65 0·30 ⩽10·35 0·35	⩾5·40 0·45			
48			9·93 0·20 7·72 0·20					*6·55* 0·20 ⩾9·35 0·35		(6·08) 0·35					
47					*7·01* 0·20 6·27 0·20	*(9·05)* 0·20 *(5·97)* 0·35				⩾8·05 0·35					
46			8·61 0·25	*(7·05)* 0·20 *(8·85)* 0·25			*(9·35)* 0·20								

Z \ D	21	22	23	24	25	26	27	28	29
60	7·0 0·2								
59		7·8 0·3 5·02 0·20	*9·50* 0·15 5·47 0·15						
58				*9·05* 0·20	5·67 0·15 8·75 0·15	*7·15* 0·20 8·80 0·20			
57				(6·45) 0·45	*8·8* 0·2 6·7 0·3		7·10 0·15 7·81 0·20		
56					*(6·8)* 0·2 ≥7·6 0·2	*(8·55)* 0·25 8·95 0·25	*5·2* 0·3 7·1 0·3		≤5·8 0·3
55			*9·05* 0·20				≥7·2 0·2	7·05 0·25	
54									
53	*9·10* 0·20								

Z \ D	33	34	35	36	37	38	39	40	41
80						6·94 0·15			*6·3* 0·2
79				⩽5·9 0·3	<5·21 0·08	<6·7 0·2	*8·0* 0·1 <6·5 0·1	*6·35* 0·15 6·32 0·15	
78						*9·5* 0·2	*6·12* 0·08 7·5 0·2	*8·2* 0·2 8·4 0·3	<6·5 0·1
77	(2·6) 0·4						(*7·8*) 0·2		7·3 0·3
76			(6·6) 0·3 (6·9) 0·3						
75					(*7·3*) 0·3				
74			(*6·25*) 0·30			(*7·15*) 0·30			
73			*7·55* 0·20	*6·03* 0·15					

Z \ *D*	40	41	42	43	44	45	46	47	48
92				4·42 0·16	7·73 0·13 5·01 0·06	6·15 0·09 5·08 0·10	7·61 0·09 5·66 0·13	5·63 0·12	7·39 0·12 6·24 0·11
91					3·47 0·13	7·14 0·11 3·58 0·07	6·08 0·09 4·43 0·11	7·03 0·13 4·22 0·12	
90				4·39 0·14	7·62 0·11 5·22 0·06	5·89 0·08 5·41 0·09	7·03 0·09 5·82 0·13	5·23 0·10 5·62 0·13	7·24 0·10 6·51 0·10
89					3·71 0·11	6·79 0·10 3·75 0·06	5·70 0·09 4·47 0·11	6·62 0·13 4·49 0·12	5·43 0·15 4·85 0·16
88				4·99 0·13	7·19 0·11 5·72 0·05	5·32 0·08 5·82 0·08	6·75 0·08 6·32 0·12	4·98 0·10	6·60 0·10 7·00 0·10
87					3·97 0·10	6·46 0·08 3·88 0·05	5·22 0·07 4·70 0·05	6·25 0·12 4·62 0·12	
86				5·09 0·09	6·66 0·08 5·85 0·04	4·58 0·07 5·84 0·07	6·55 0·07 6·53 0·12	4·40 0·05 6·29 0·08	6·33 0·03
85					4·08 0·08	5·90 0·06 4·07 0·04	4·59 0·06 4·56 0·04	5·86 0·12 4·64 0·12	4·64 0·14
84		6·65 0·18 4·84 0·13	7·66 0·10 5·06 0·07	4·85 0·03 5·24 0·04	5·72 0·03 5·86 0·02	4·31 0·05 5·79 0·05	5·91 0·06 6·63 0·12	4·10 0·04 6·49 0·04	5·78 0·03
83			3·67 0·07	*7·44* 0·05 3·73 0·05	[4·67] 0·06 4·53 0·06	5·10 0·04 4·39 0·04	4·38 0·02 4·99 0·03	5·07 0·12 4·89 0·12	4·24 0·12
82	6·51 0·15	6·40 0·18 6·39 0·18	*8·10* 0·10 6·89 0·15	*6·73* 0·01 7·46 0·02	*7·38* 0·01 8·05 0·02	*3·87* 0·05 8·09 0·05	5·24 0·06 8·49 0·12	3·78 0·04 8·39 0·05	5·17 0·03
81		*8·80* 0·20	*6·52* 0·15 6·23 0·15	*7·60* 0·20 < 6·8 0·2	*6·16* 0·15 7·08 0·18	6·79 0·03	3·83 0·03	4·84 0·11	3·88 0·13

Z D	49	50	51	52	53	54	55	56	57
98		4·80 0·15							
97	3·32 0·17		4·41 0·14						
96		5·22 0·14	5·72 0·15 5·53 0·15	6·51 0·07 5·81 0·08					
95	3·94 0·16		4·95 0·13	5·41 0·14 4·65 0·14	6·23 0·15 4·82 0·15				
94		5·79 0·14	5·50 0·15 6·10 0·15	6·42 0·05 6·35 0·05	5·71 0·12	6·06 0·14			
93	4·20 0·16	5·81 0·10 4·86 0·09	6·91 0·16 5·34 0·13	5·19 0·14 5·08 0·14	6·17 0·15 5·22 0·14				
92	5·97 0·14	6·74 0·14 6·53 0·14	5·15 0·15 6·30 0·16	6·43 0·04	5·45 0·12	6·03 0·13	4·72 0·13		
91	4·50 0·16		5·61 0·11	5·38 0·14 5·83 0·14					
90	5·48 0·13 6·94 0·13	6·68 0·14	4·96 0·16 6·4 0·3	*6·35* 0·04	5·16 0·11	6·01 0·14			
89	6·35 0·13 4·87 0·16	5·05 0·10 5·55 0·11							
88	5·07 0·13	6·33 0·14	4·37 0·16	6·28 0·11					
87	5·02 0·16								

Z \ *D*	40	41	42	43	44	45	46	47	48
98									4·58 0·13
97									
96									5·02 0·11
95									
94					4·70 0·08		5·01 0·15	5·98 0·14	7·49 0·14 5·54 0·10
93						3·14 0·08		3·97 0·14	

APPENDIX II

In this appendix further consideration is given to the very few instances in which the binding energies listed in Appendix I have not been calculated on the assumption that the ‘ maximum ’ reaction energies revealed by experiment are in fact the energies of ground-to-ground state transition in all cases. In these few instances that assumption was found to run counter to other evidence, and adjustments were made which are detailed below. Certain other apparent discrepancies are referred to briefly at the end of the appendix.

$B_n\binom{223}{88}$, $B_p\binom{224}{89}$, $B_n\binom{227}{90}$, $B_p\binom{228}{91}$: The values of these four binding energies depend directly on $T_\alpha\binom{223}{88}$. We have been led to suspect that the experimental ‘ maximum ’ disintegration energy of ${}^{223}_{88}$AcX, which is given in the literature as 5·82 MeV, is not the true disintegration energy, ground state to ground state, but rather that the true value is 6·05±0·08 MeV. The basis of calculation is as follows :

$$\begin{aligned}
T_\alpha\binom{223}{88}
&=B_n\binom{232}{90}+T_\alpha\binom{232}{90}-T_\alpha\binom{231}{91}-T_{\beta}^{-}\binom{231}{90}+T_{\beta}^{-}\binom{228}{88}+T_{\beta}^{-}\binom{228}{89}\\
&\quad-T_{\beta}^{-}\binom{227}{89}+T_\alpha\binom{228}{90}+T_\alpha\binom{224}{88}+T_\alpha\binom{220}{86}+T_\alpha\binom{216}{84}\\
&\quad-T_\alpha\binom{227}{90}-T_\alpha\binom{219}{86}-T_\alpha\binom{215}{84}+T_{\beta}^{-}\binom{212}{82}-T_{\beta}^{-}\binom{211}{82}\\
&\quad+T_\alpha\binom{212}{83}-T_\alpha\binom{211}{83}+T_{\beta}^{-}\binom{208}{81}-T_{\beta}^{-}\binom{207}{81}-B_n\binom{208}{82}.
\end{aligned}$$

Experimental values being inserted for the 21 quantities represented on the right hand side of this equation we obtain $T_\alpha\binom{223}{88}=6{\cdot}05\pm0{\cdot}08$ MeV as already stated. For this calculation concordant values of $B_n\binom{208}{82}$ are available from the results of (d, p), (n, γ) and (γ, n) reactions with the appropriate lead isotopes, but $B_n\binom{232}{90}$ is known only as a result of a determination of the threshold energy for the (γ, n) reaction with ${}^{232}_{90}$Th. Although it would be possible to assume that the discrepancy in the closed cycle (of 22 terms) derives from this determination alone, that view does not appear as plausible as the one here put forward. Alternatively, the discrepancy might have been assumed to derive from the determination of any one or more of the disintegration energies involved in the cycle : we have fixed upon the one which appears as the most likely source.

With ${}^{223}_{88}$AcX, rather than with any other of the bodies concerned, there appears at first sight to be the greatest difficulty in constructing a satisfactory disintegration scheme without some increase in the accepted value of the total energy of disintegration.* Adopting this assumption, not only the four last-nucleon binding energies listed above, but also the values of $B_p\binom{227}{90}$, $B_n\binom{227}{89}$, $B_p\binom{227}{89}$ and $B_n\binom{227}{88}$ reflect the admittedly arbitrary adjustment that we have made.

$B_n\binom{210}{83}$: It was first clearly pointed out by Huizenga, Magnusson, Simpson and Winslow (1950) that the value of this binding energy is some 0·4 MeV greater than the lower limit given by investigations on the (n, γ) and (d, p) reactions with ${}^{209}_{83}$Bi. These two investigations are mutually concordant in fixing the lower limit at 4·16±0·02 MeV. The calculation (slightly simpler than that of Huizenga *et al.*) which shows that this value is in fact significantly low is based on the closed cycle

$$B_n\binom{210}{83}+B_n\binom{209}{82}+B_n\binom{208}{82}+B_n\binom{207}{82}+T_\beta^-\binom{210}{83}+T_\beta^-\binom{209}{82}$$
$$+T_\alpha\binom{210}{84}-2T_\beta^-\binom{1}{0}+N\binom{4}{2}-2N\binom{1}{1}-2N\binom{1}{0}=0.$$

Inserting experimental values for all quantities except the first, we have $B_n\binom{210}{83}=4{\cdot}67\pm0{\cdot}06$ MeV. It should be noted (as Huizenga *et al.* noted) that the discrepancy could alternatively have been attributed to the interpretation of the results of the investigation of the (d, p) reaction with ${}^{208}_{82}$Pb, giving $B_n\binom{209}{82}$, but along with these authors we consider the present attribution the more likely.

Of the binding energies listed in Appendix I only $B_n\binom{210}{84}$ and $B_p\binom{210}{84}$ derive directly from $B_n\binom{210}{83}$.

$B_p\binom{40}{18}$: There have been two determinations of this binding energy on the basis of the threshold energy of the (γ, p) reaction with ${}^{40}_{18}$A.

In 1950 Haslam, Katz, Moody and Skarsgard reported a threshold energy of 14·2±0·2 MeV: later Wilkinson and Carver (1951) reported 10·8±0·1 MeV for the same reaction. First inclinations would be to accept the lower value as correct, on the general consideration that such a threshold energy is at best an upper limiting value, but detailed considerations appear to make such acceptance impossible. We have

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* Thanks are due to J. Kyles, C. G. Campbell and W. J. Henderson for permission to use their value of $T_\beta^-\binom{228}{89}$ prior to publication, and to M. A. S. Ross and B. F. Bayman for discussions on possible disintegration schemes to represent the complete α-disintegration of ${}^{223}_{88}$AcX.

$$B_p\binom{40}{18}=B_n\binom{40}{19}-T_\beta^-\binom{40}{18}+T_\beta^-\binom{39}{18}+T_\beta^-\binom{39}{17}-T_\beta^-\binom{1}{0},$$

and with listed values of the relevant quantities we obtain $B_p\binom{40}{18}=12{\cdot}44\pm0{\cdot}15$ MeV. Here the greatest admitted uncertainty in the experimentally determined energies is in $T_\beta^-\binom{40}{18}$ but no amount of adjustment can reduce the calculated value of $B_p\binom{40}{18}$ by more than 0·15 MeV in respect of this term. Again, even if there is some gross error of interpretation in respect of the long-lived ${}^{39}_{18}\mathrm{A}$, this species is surely β-active and $B_p\binom{40}{18}$ would not be diminished by more than 0·57 MeV if the activity were regarded as marginal. Finally, $B_n\binom{40}{19}$ is known only from investigations of the (d, p) and (n, γ) reactions with ${}^{39}_{19}\mathrm{K}$; any failure of interpretation on correction would yield an even greater value for $B_p\binom{40}{18}$ than we have already obtained. In balance there appears to be good reason for accepting our calculated value, in preference to either value deduced from investigations of the (γ, p) reaction—at least until further experimental evidence is available.

$B_n\binom{93}{42}$, $B_p\binom{93}{42}$: The difference between these two binding energies involves the unknown energy of β-disintegration of ${}^{92}_{41}\mathrm{Nb}$. In this case $B_n\binom{93}{42}$ is known, as a lower limiting value, from the (d, p) reaction energy with ${}^{92}_{42}\mathrm{Mo}$ but $B_p\binom{93}{42}$ is not directly determined. We have, in fact,

$$T_\beta^-\binom{92}{41}=B_n\binom{93}{41}+Q_\gamma\binom{93}{41}-B_n\binom{93}{42}+T_\beta^-\binom{1}{0},$$

and, with accepted values inserted, obtain $T_\beta^-\binom{92}{41}=-0{\cdot}08\pm0{\cdot}35$ MeV. This calculated disintegration energy is an upper limit, both because of the basis of determination of $B_n\binom{93}{42}$ already mentioned, and also because the experimental value of $B_n\binom{93}{41}$ is itself an upper limit, deriving from information concerning the (γ, n) reaction with ${}^{93}_{41}\mathrm{Nb}$. As it stands it is not significantly negative, but because of its character as a (double) upper limit, and because of a preliminary report which would suggest that our listed value for $B_n\binom{93}{42}$ is in fact too low by some 0·4 MeV (Shull and

McFarland 1952), this formal discrepancy is worth remarking. If it should be substantiated it would indicate that ${}^{92}_{42}\mathrm{Mo}$ is capture-active. This may not appear very likely (although the position in respect of this lightest isotope of molybdenum is already anomalous in that its isotopic abundance is unusually high)— but attention may properly be drawn to the recent finding of Fremlin and Walters (1952) of “ clear evidence of activity ” in a spectroscopically pure sample of molybdenum investigated by the nuclear emulsion method. It remains to be seen, first, whether this finding is confirmed, and secondly, if it is, whether the observed activity is such as might be explained by the capture instability of ${}^{92}_{42}\mathrm{Mo}$.

$B_n\binom{239}{92}$: A lower limiting value for this binding energy has been determined from the $(d,\ p)$ reaction energy with ${}^{238}_{92}\mathrm{U}$. Starting from $B_n\binom{207}{82}$, the value of which is known without such uncertainty from investigations of suitable $(d,\ p)$, $(n,\ \gamma)$, $(\gamma,\ n)$ and $(d,\ t)$ reactions, another value for $B_n\binom{239}{92}$ may be calculated on the basis of the closed cycle result

$$\begin{aligned}
B_n\binom{239}{92}
&= B_n\binom{207}{82} + T_\alpha\binom{238}{92} + T_{\beta}^{-}\binom{234}{90} + T_{\beta}^{-}\binom{234}{91} + T_\alpha\binom{234}{92} + T_\alpha\binom{230}{90}\\
&\quad + T_\alpha\binom{226}{88} + T_\alpha\binom{222}{86} + T_\alpha\binom{218}{84} + T_{\beta}^{-}\binom{214}{82} + T_{\beta}^{-}\binom{214}{83}\\
&\quad + T_\alpha\binom{214}{84} + T_{\beta}^{-}\binom{210}{82} + T_{\beta}^{-}\binom{210}{83} + T_\alpha\binom{210}{84} - T_{\beta}^{-}\binom{239}{92}\\
&\quad - T_{\beta}^{-}\binom{239}{93} - T_\alpha\binom{239}{94} - T_\alpha\binom{235}{92} - T_{\beta}^{-}\binom{231}{90} - T_\alpha\binom{231}{91}\\
&\quad - T_{\beta}^{-}\binom{227}{89} - T_\alpha\binom{227}{90} - T_\alpha\binom{223}{88} - T_\alpha\binom{219}{86} - T_\alpha\binom{215}{84}\\
&\quad - T_{\beta}^{-}\binom{211}{82} - T_\alpha\binom{211}{83} - T_{\beta}^{-}\binom{207}{81}.
\end{aligned}$$

When the adjusted value, $T_\alpha\binom{223}{88} = 6{\cdot}05 \pm 0{\cdot}08$ Mev is used together with accepted values of the other disintegration energies in this equation, we obtain $B_n\binom{239}{92} = 4{\cdot}72 \pm 0{\cdot}13$ Mev, in agreement, within the stated limits of probable error, with the direct experimental lower limit $B_n\binom{239}{92} \geqslant 4{\cdot}63 \pm 0{\cdot}15$ Mev. For sake of consistency of the table we have preferred the ‘ calculated ’ to the ‘ direct ’ value in Appendix I.

It is to be noted here that if we had accepted the direct value of $B_n\binom{239}{92}$ as primary datum we should have obtained $T_\alpha\binom{223}{88}=6{\cdot}14\pm0{\cdot}18$ Mev, not so closely defined as the value $6{\cdot}05\pm0{\cdot}08$ Mev based upon $B_n\binom{232}{90}$, but even further removed from the experimental maximum disintegration energy $5{\cdot}82$ Mev. This consideration makes it the more certain that the true value of this disintegration energy (or one or more of the disintegration energies in the sequence from $^{231}_{90}$UY to $^{207}_{82}$Pb) is higher than the experimentally determined value.

$B_n\binom{238}{92}$: Concordant values of this binding energy are available as the result of investigations of the $(d,\,p)$ reaction with $^{237}_{92}$U and the $(\gamma,\,n)$ reaction with $^{238}_{92}$U. A 'calculated' value may be based on the adjusted value of $B_n\binom{210}{83}$ using the result

$$\begin{aligned}
&B_n\binom{238}{92}\\
&=B_n\binom{210}{83}-T_\alpha\binom{241}{94}+T_\beta^-\binom{241}{94}+T_\alpha\binom{241}{95}+T_\alpha\binom{237}{93}+T_\beta^-\binom{233}{91}\\
&\qquad+T_\alpha\binom{233}{92}+T_\alpha\binom{229}{90}+T_\beta^-\binom{225}{88}+T_\alpha\binom{225}{89}+T_\alpha\binom{221}{87}\\
&\qquad+T_\alpha\binom{217}{85}+T_\beta^-\binom{213}{83}+T_\alpha\binom{213}{84}+T_\beta^-\binom{209}{82}-T_\alpha\binom{238}{92}\\
&\qquad-T_\beta^-\binom{234}{90}-T_\beta^-\binom{234}{91}-T_\alpha\binom{234}{92}-T_\alpha\binom{230}{90}-T_\alpha\binom{226}{88}\\
&\qquad-T_\alpha\binom{222}{86}-T_\alpha\binom{218}{84}-T_\beta^-\binom{214}{82}-T_\beta^-\binom{214}{83}-T_\alpha\binom{214}{84}\\
&\qquad-T_\beta^-\binom{210}{82}.
\end{aligned}$$

On this basis we obtain $B_n\binom{238}{92}=6{\cdot}03\pm0{\cdot}13$ Mev, in agreement with the directly determined $B_n\binom{238}{92}=5{\cdot}97\pm0{\cdot}10$ Mev well within the combined limits of error of the two determinations. As before, we use the calculated value in Appendix I for sake of general consistency.

It should be noted here that the agreement just recorded provides further support, if that were needed, for the adjustment of $B_n\binom{210}{83}$ from the experimental $B_n\binom{210}{83}\geqslant4{\cdot}16\pm0{\cdot}02$ Mev to the calculated $B_n\binom{210}{83}=4{\cdot}67\pm0{\cdot}06$ Mev. In this connection we have assumed $T_\beta^-\binom{225}{88}=0{\cdot}20\pm0{\cdot}05$ Mev and $T_\beta^-\binom{213}{83}=1{\cdot}50\pm0{\cdot}06$ Mev.

$B_n\binom{233}{90}$: The experimental value for this binding energy, $B_n\binom{233}{90} \geqslant 4{\cdot}9 \pm 0{\cdot}2$ MeV, derives from the reaction energy of the (d, p) reaction with $^{232}_{90}$Th. A calculated value, based upon $B_n\binom{209}{82}$, may be obtained from the closed cycle result

$$\begin{aligned} B_n\binom{233}{90} &= B_n\binom{209}{82} + T_\alpha\binom{232}{90} + T_\beta^-\binom{228}{88} + T_\beta^-\binom{228}{89} + T_\alpha\binom{228}{90} + T_\alpha\binom{224}{88} \\ &\quad + T_\alpha\binom{220}{86} + T_\alpha\binom{216}{84} + T_\beta^-\binom{212}{82} + T_\beta^-\binom{212}{83} + T_\alpha\binom{212}{84} \\ &\quad - T_\beta^-\binom{233}{90} - T_\beta^-\binom{233}{91} - T_\alpha\binom{233}{92} - T_\alpha\binom{229}{90} - T_\beta^-\binom{225}{88} \\ &\quad - T_\alpha\binom{225}{89} - T_\alpha\binom{221}{87} - T_\alpha\binom{217}{85} - T_\beta^-\binom{213}{83} - T_\alpha\binom{213}{84}. \end{aligned}$$

Strictly, $B_n\binom{209}{82}$ is known only as a lower limiting value from the (d, p) reaction energy with $^{208}_{82}$Pb, but if we accept this value as the true value we obtain $B_n\binom{233}{90} = 5{\cdot}16 \pm 0{\cdot}11$ MeV. It is not implausible to regard this as confirming $4{\cdot}9 \pm 0{\cdot}2$ MeV as a true value, not very precisely known; moreover, in view of agreements noted above, there is very little that can be done by way of adjustment here (except possibly to increase $T_\beta^-\binom{233}{90}$ very slightly). For lack of any clear evidence of discrepancy we have accepted $B_n\binom{209}{82} = 3{\cdot}87 \pm 0{\cdot}05$ MeV and $B_n\binom{233}{90} = 5{\cdot}16 \pm 0{\cdot}11$ MeV in Appendix I.

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Mesonic Interpretation of Nuclear Properties

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§ 1. Introduction

1.1

It is known that charged and neutral π-mesons are strongly coupled to nucleons, a coupling which reveals itself in the production cross-sections for π-mesons in nucleon–nucleon and photon–nucleon collisions (for references see the accompanying paper by Gunn (1953)). If nuclear forces are explained as due to the exchange of virtual π-mesons the order of magnitude of the meson–nucleon coupling (as revealed by the production experiments) is such as to give the right order of magnitude of the strength of nuclear forces. In addition the masses of the π-mesons give correctly the range of nuclear forces. These properties (though the mass–range relation may be deduced by general arguments (Rosenfeld 1948 a)) are derived from a formalism of the interaction of π-mesons with nucleons originally given in this context by Yukawa (1935). The formalism is set out in § 1.2 below and unless otherwise signified the phrase π-meson (or meson) field theory will henceforth refer to this formalism. While π-meson field theory is qualitatively successful in this elementary way, it is not successful in the quantitative explanation of nuclear forces or other nuclear properties, such as the magnetic moments of protons and neutrons. The mathematical methods so far used to derive nuclear properties from π-meson field theory are probably inadequate ; and it is not known whether the solutions given by these methods closely approach the correct solutions of the basic equations of π-meson–nucleon field theory. So it is not known whether there is a fundamental fault in the present theory of the interaction of π-mesons with nucleons, or whether the failure to get quantitative agreement is due to poor mathematics. As a working assumption, it is generally taken that the latter is the case, and a main effort in π-meson field theories is directed to the improvement of the calculations. This article will for the most part record these efforts and their degree of success.

An isolated proton (or neutron) will be accompanied by a charged and neutral π-meson field, which modifies the proton (or neutron) properties in an electromagnetic field. The observed electromagnetic properties of single nucleons, and the methods of calculating electromagnetic properties from meson field theory are discussed in § 2. In § 3 the attempts at calculating the forces between two nucleons from π-meson field theory are reviewed. In § 4 many body systems—that is to say, in practice, heavy nuclei—are considered. In such systems, besides the two body forces of § 3, meson theory predicts additional ‘ many body forces ’. It is extremely difficult to estimate the many body forces and thus discussions of nuclear properties (for instance the saturation property) are usually based on the two body forces alone. In connection with saturation a (' non-linear ') modification of π-meson field theory is reported.

Besides the π-meson there are probably other particles having a direct interaction with protons and neutrons. Such particles are the ξ-meson (Danysz *et al.* 1952) of about 600 electron masses, the κ- and ζ-mesons (O'Ceallaigh 1951) of about 1300 electron masses and the V-mesons (Armenteros *et al.* 1951, Seriff *et al.* 1950) of about 2100 electron masses. So little is known about the relation of these particles to protons and neutrons that it is impossible to estimate their effect on nuclear forces. This situation is mitigated by the large mass of the heavy mesons, which results in any nuclear forces due to them being of smaller range than that of the π-meson force (Rosenfeld 1948 a). This short range force, if it exists, is unimportant for some processes (for example, low energy nucleon–nucleon scattering). For others the possibility of its existence may have to be ignored.*

The mesonic interpretation of nuclear properties is much older than the identification (Lattes, Muirhead, Occhialini and Powell 1947) of the π-meson, and dates from Yukawa's (1935) postulation of the π-meson to explain nuclear forces. Since it is now almost certain that the π-meson has spin 0 (Durbin *et al.* 1951, Clark *et al.* 1951) the possible explanation of nucleon properties by spin 1 mesons, much discussed in earlier days (see Fröhlich *et al.* 1938, Bethe 1940) can probably be disregarded, and will not be discussed.

1.2

The formalism for the meson field in interaction with protons and neutrons is now to be given (Yukawa 1935, Kemmer 1938 a, Pauli 1941, Wentzel 1949). Let $|\rangle$ be a *ket* describing a nucleon–meson system. Then $|\rangle$ obeys a Schrödinger equation given by

$$i\hbar\frac{d}{dt}|\rangle=H|\rangle . \qquad (1.21)$$

The Hamiltonian H is given by

$$H=\int_{-\infty}^{\infty}(H_1+H_2+H'+H'')d^3x, \qquad (1.22)$$

$$H_1=-\Pi\left(\alpha_i\frac{\partial}{\partial x_i}+i\beta\kappa\right)\Psi, \qquad (1.23)$$

$$H_2=\tfrac{1}{2}\sum_{\lambda=1}^{4}\left(\sum_{j=1}^{3}\left(\frac{\partial}{\partial x_j}\phi_\lambda\right)^2+\mu^2\phi_\lambda^2+\pi_\lambda^2\right). \qquad (1.24)$$

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* Both Jastrow (1951) and Levy (1952 b) introduce an infinite repulsive potential in the nucleon–nucleon force at distances of the order of the nucleon Compton wavelength. It is possible that heavy mesons contribute to such a short range repulsive potential, and that their total contribution to nuclear forces, within the energy range at present contemplated, is adequately described by such a potential.

H' takes on one of four forms for spin 0 mesons.

$$H_a'=\sum_{\lambda=1}^{4} ig_\lambda \Pi\beta\tau_\lambda\Psi\phi_\lambda, \quad . \ . \ . \ . \ . \ . \ . \ . \tag{1.25 a}$$

$$H_b'=\sum_{\lambda=1}^{4} \frac{i}{\mu} f_\lambda \Pi\left(\pi_\lambda-\alpha_i\frac{\partial\phi_\lambda}{\partial x_i}\right)\tau_\lambda\Psi, \quad . \ . \ . \ . \ . \tag{1.25 b}$$

$$H_A'=\sum_{\lambda=1}^{4} G_\lambda \Pi\beta\gamma_5\phi_\lambda\tau_\lambda\Psi, \quad . \ . \ . \ . \ . \ . \ . \tag{1.25 A}$$

$$H_B'=\sum_{\lambda=1}^{4} \frac{1}{\mu} F_\lambda \Pi\beta\gamma_5\left(\pi_\lambda-\alpha_i\frac{\partial\phi_\lambda}{\partial x_i}\right)\tau_\lambda\Psi. \quad . \ . \ . \tag{1.25 B}$$

H'' is the coupling to the electromagnetic field, and is given in the appendix.

Units in which $\hbar=c=1$ have been taken. The neutron and proton masses have been taken equal. Ψ is the 8-component spinor for the proton and neutron. $\Pi(=i\Psi^*)$ is the canonically conjugate quantity. The components of the spinors are $\Psi_{\varrho\sigma}$, $\Pi_{\varrho\sigma}$, where σ runs from 1 to 2 and is the isotopic spin, or charge, variable and ρ runs from 1 to 4. Π and Ψ obey the commutation relations given in the appendix.

The β, α_i are the usual Dirac matrices

$$\gamma_5=\gamma_1\,\gamma_2\,\gamma_3\,\gamma_4$$

where $\gamma_4=\beta$ and $\gamma_i=-i\beta\alpha_i$ for $i=1, 2, 3$.

The ϕ_1, ϕ_2, ϕ_3, ϕ_4 are real meson field variables, and π_1, π_2, π_3, π_4, are the canonically conjugate quantities. The commutation rules are given in the appendix. ϕ_1 and ϕ_2 are the charged meson field variables given in terms of the usual complex meson field variables by $\phi_1=(\phi+\phi^*)/\sqrt{2}$, $\phi_2=i(\phi-\phi^*)/\sqrt{2}$. ϕ_3 and ϕ_4 are meson field variables for two different kinds of neutral mesons.

These two latter variables are distinguished by their coupling to the nucleon field. In the coupling term H', τ_1, τ_2, τ_3, τ_4 are the usual isotopic spin matrices $\tau_1=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$, $\tau_2=\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}$, $\tau_3=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$, $\tau_4=1=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$, acting on the isotopic spin variables of the spinors Π and Ψ. As can be seen from the form of H', ϕ_3 is associated with τ_3 and ϕ_4 with τ_4. H_a', H_b', H_A', H_B' are the scalar and vector coupling of scalar spin zero mesons and pseudoscalar and pseudovector coupling of pseudoscalar spin zero mesons respectively. In what follows these may be referred to as the s–s, s–v, ps–ps and ps–pv couplings respectively. It is assumed that positive and negative charge are equivalent and thus $c_1=c_2$ (c is a symbol for which may be substituted either f, g, F or G).

The coupling in which $c_1=c_2=c_3$, $c_4=0$ is the symmetric theory of Kemmer (1938 b). If V is the accurate potential between two nucleons, the symmetric theory gives, for the isotopic spin dependence of V,

$$V=A+B\sum_{i=1}^{3}\tau_i^{(1)}\,\tau_i^{(2)}.$$

This gives the nucleon–nucleon interaction charge independent in the sense that proton–proton and neutron–neutron (mesonic) forces are equal and equal to the proton–neutron force, when the latter system is in odd space–spin state. This 'charge independence' hypothesis agrees well with the data on low energy nucleon–nucleon scattering. Another argument in favour of the symmetric theory is that with this coupling the total isotopic spin commutes with the Hamiltonian (1.22). Thus the total isotopic spin is a constant of the motion.

Anderson, Fermi *et al.* (1952 c) have made an analysis into phase shifts of S and P waves of the Chicago π-meson on proton scattering experiments (Anderson *et al.* 1952 a, b, c, Fermi *et al.* 1952). With the assumption that the total isotopic spin is a constant of the motion these authors fit nine experimental points with six phase shifts. Though the experimental evidence is not yet sufficient to be conclusive, it certainly seems to indicate that the total isotopic spin commutes with the Hamiltonian, as in the symmetrical theory.

Besides the symmetrical another possible theory is one in which all the neutral mesons have τ_4 coupling. That is to say $c_3=0$, $c_4\neq 0$. With such a coupling the isotopic spin is not a constant of the motion, so the analysis of Anderson, Fermi *et al.* (1952 c) of the π-meson scattering experiments does not hold. However, the evidence for total isotopic spin being a constant of the motion is not yet good enough to rule out the τ_4 coupling of neutral mesons. The τ_4 coupling (when charged mesons are also present) does not give a potential of the form (1.28), nor does it necessarily give a nucleon–nucleon potential satisfying the charge independent hypothesis.

As well as the pure τ_3 coupling (c_4 zero) or pure τ_4 coupling (c_3 zero) of neutral mesons, a coupling is envisigable in which both kinds of neutral mesons exist. In this neither the third nor the fourth components of the coupling constant are zero.

Despite the uncertainty in the results from the production experiments, and the limited number of the π-meson scattering results from the Chicago synchrotron, the symmetrical theory seems a plausible working hypothesis. Much, though certainly not exclusive, attention has been paid to it by authors on nuclear properties.

In the first two couplings of (1.25) the meson field has scalar transformation properties, in the last two, pseudoscalar. Experiments (Panofsky *et al.* 1951) on the absorption of negative π-mesons in deuterium provide some indication that the negative π-meson is pseudoscalar. From the result of this experiment Tamor and Marshak (1950) show from general invariance arguments that the π-meson is pseudoscalar unless it is absorbed by the deuteron from the P-orbit, instead of the S-orbit. Perturbation calculations indicate that the absorption from the P-orbit is much slower than transitions from the P-orbit to the S-orbit, thus implying that nearly all absorption is from the S-orbit, and that the π^--meson is pseudoscalar. However, it is well known that perturbation calculations in meson theory are untrustworthy, so it is still an open question whether the π^--meson is scalar or pseudoscalar, though the indication is certainly in favour of the latter.

It is, of course, possible that H', instead of being a single one of the couplings (1.25 a, b, A, B) should be a mixture of two or more. Included in this conception, for example, is a coupling in which the charged mesons are pseudoscalar and the neutral scalar. Such a mixture, though apparently unaesthetic, is certainly permissible, and a final decision can only be given by experiment. A large number of mixtures of this kind are possible, but in general they have not been considered, and investigations in the past have been largely confined to pure interactions; that is (1.25 a), (1.25 b), (1.25 A) or (1.25 B).

1.3

A formal difficulty arises in the solution and interpretation of eqn. (1.21). This equation can ideally be solved for a particular system by a perturbation expansion in the coupling constant (in practice, of course, only the very few first terms of this expansion can be written down). If this solution is interpreted naively certain physical quantities of the system turn out to be infinite. For example, the second term in the lower series for proton–proton scattering is infinite. However, for the interactions (1.25 a), (1.25 A) the divergences can be absorbed into the masses and the coupling constants, by a consistent renormalization of these quantities, leaving all physically observable quantities finite (Schwinger 1948 a, b, Schwinger 1949, Ito, Koba and Tomonaga 1948, Koba and Tomonaga 1948, Dyson 1949 a, b). (This renormalization actually involves the introduction of a term $\lambda(\phi_1^2+\phi_2^2)^2+\lambda_3\phi_3^4+\lambda_4\phi_4^4$ into the Lagrangian, with a corresponding term in the Hamiltonian. λ, λ_3, λ_4 are arbitrary constants and can be taken arbitrarily small, and indeed zero. Thus if the λ are taken zero, the Hamiltonian (1.22) remains the same, and the (1.25a) and (1.25 A) theories can be made finite.) The divergence difficulty is probably deep-rooted in the theory, and is not just a result of the perturbation expansion. Thus the theories with coupling (1.25 b, B) are not at present so satisfactory as the other two. Despite this the couplings (1.25 b, B) can be, and are used because in the perturbation series the first non-vanishing term for a process is usually finite, and in other methods a high frequency cut-off can be invoked to eliminate divergences.

Thus in π-meson field theory all the couplings (1.25 a, b, A, B) must be considered until such time as a comparison of theory and experiment distinguishes one as correct. It is always borne in mind that none may be correct, and a mixture may have to be taken, or some very much more drastic change made in the formalism.

§ 2. ELECTROMAGNETIC PROPERTIES OF PROTONS AND NEUTRONS

Apart from the scattering and photo-production and absorption of π-mesons (Gunn 1953) the only way in which the mesonic properties of single nucleons not interacting with other nucleons at present become manifest is through their electromagnetic properties. The electromagnetic properties to be explained are the magnetic moments of the proton and neutron and the neutron–electron interaction. In § 2.1 magnetic moments are considered and in § 2.2 the neutron–electron interaction.

2.1

The magnetic moment of the proton in the direction of its spin is 2·79 Bohr nuclear magnetons and that of the neutron in the direction of its spin is $-1{\cdot}91$ Bohr nuclear magnetons. The proton and neutron are assumed to be described by eqns. (1.21), (1.22), (1.23), (1.24), (1.25). If the coupling constant (g, f, G or F) is put equal to 0, so that H' is zero and the proton and neutron are unassociated with the meson field, then the proton and neutron obey the Dirac equation for particles of charge e and 0. Thus an 'unassociated' or 'bare' proton would exhibit a magnetic moment (apart from very small radiative corrections) of one 'bare' nuclear magneton,* and a 'bare' neutron a magnetic moment zero. The change in the magnetic moments is then to be attributed to the action of the meson field coupling.

(*a*) One method of attempting to solve eqn. (1.21) for the magnetic moment of the nucleon is the perturbation method (Heitler 1944 a) of expanding the solution as a power series in the meson–nucleon coupling constant. Especially as only the first one or two terms in the series can be evaluated, the effectiveness of the method depends on H' being only a small perturbation term. As the meson–nucleon coupling constant is quite large, it is thus by no means evident that the perturbation method gives a good approximation to the solution of (1.21). In default of better methods, the perturbation method is the one most commonly used to solve eqn. (1.21) for nuclear properties, and it is only very rarely that an estimate of the error involved in its use can be given. Such an estimate cannot be given in the magnetic moment case.

The perturbation series for the magnetic moment consists of terms of zero, second, fourth and all even orders in the meson–nucleon coupling constant. For the s–v and ps–pv theories the fourth order and higher order terms diverge and the theory cannot be renormalized in these higher orders. So perturbation theory only gives a means of calculating the magnetic moment in second order for these couplings (except by putting in an arbitrary high frequency cut-off, or using some other ad hoc procedure). The s–s and ps–ps couplings (1.25 *a*, *A*) can be renormalized to give finite results in all orders. The zero order term gives just the Dirac magnetic moment (that is to say the magnetic moment as if the nucleon obeyed the Dirac equation without any coupling to the meson field) of one nuclear magneton for the proton and zero for the neutron. Taking the magnetic moment of the proton first this term corresponds to a process in which the original proton interacts with the electromagnetic field. This is shown in the Feynman diagram (Dyson 1949 a), fig. 1 (*a*), where the full line represents the proton, the wavy line the photon (electromagnetic field) and the arrow follows the charged particle. The second order term corresponds to a process in which the proton comes in, emits a 'virtual' meson and a 'virtual' nucleon, one of which is charged and interacts with the electromagnetic field. The 'virtual' meson and nucleon are then reabsorbed to give the proton. These processes are represented in the Feynman diagrams of figs. 1 (*a*) and 1 (*b*). The dotted lines are mesons and the arrow follows the charged particle. In 1 (*b*) the emitted meson is charged, and in 1 (*c*) it is neutral. There are a large number of Feynman diagrams corresponding to contributions to the fourth order term and a typical one is shown in fig. 1 (*d*).

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* By 'bare' nucleon magneton is meant the Dirac magnetic moment associated with the mass, κ', of the 'bare' particle. It should be noted that it is not this (unrenormalized) 'bare' mass κ' which appears in the Hamiltonian (1.23), but the (renormalized) real mass κ. This is quite a valid formalism so long as it is understood that there are actually certain terms in the Hamiltonian which are not written down and which just serve to eliminate the infinities (Gupta 1951).

The 'bare' nucleon is certainly a myth, and, in view of the strength of the meson–nucleon coupling constant, it is quite likely not to be a helpful mathematical concept; the observable nucleon consists of a cloud of mesons and nucleon–anti-nucleon pairs.

Fig. 1

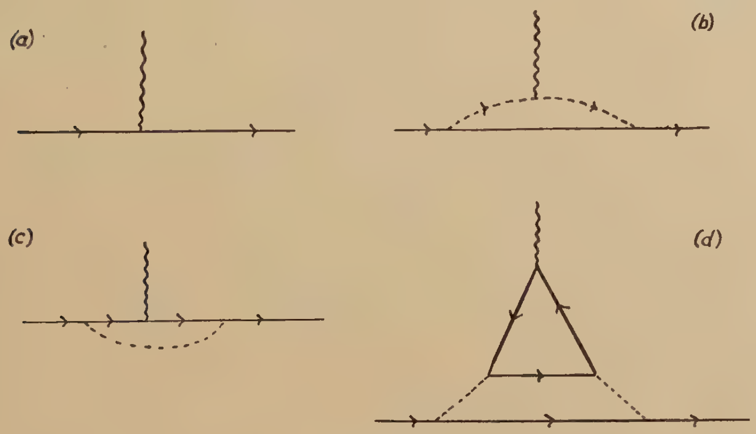
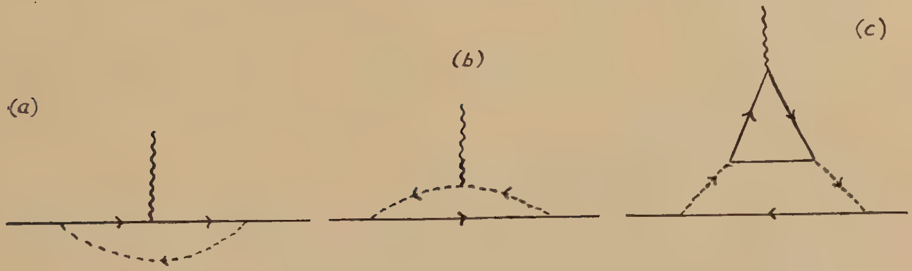


Fig. 2



In the expansion for the magnetic moment of the neutron the zero order term gives no contribution, as the neutron by itself does not interact with the electromagnetic field. The second order terms are shown in figs. 2 (*a*), 2 (*b*) and are those in which a charged meson and a charged nucleon are emitted. Figure 2 (*c*) gives an example of a fourth order term.

Many authors have evaluated the second order terms both for scalar mesons (Case 1949 a) and pseudoscalar mesons (Luttinger 1949, Slotnick and Heitler 1949, Case 1949 a, Sawada 1949). By the equivalence theorem (Case 1949 b) the second order terms in s–v (ps–pv) coupling may be directly obtained from the second order terms in s–s (ps–ps) coupling. Nakabayasi and Sato (1951) have worked out the fourth order term for pseudoscalar mesons with pseudoscalar coupling and Thellung (1952) the same coupling with $G_1=G_2=G_3=0$. The complexity of analysis increases rapidly with the order in the coupling constant; thus so far sixth and higher order terms have not been evaluated.

In the case of pseudoscalar mesons with pseudoscalar coupling the magnetic moments in nuclear magnetons up to fourth order (with the fourth order results those of Nakabayasi and Sato*) are, with $G_1=G_2=G$

$$\begin{aligned}\mu_{\mathrm{P}}=1&+\frac{1}{\pi}\left(0{\cdot}348\,\frac{G^2}{4\pi}-0{\cdot}237\left(\frac{G_3{}^2}{4\pi}+\frac{G_4{}^2}{4\pi}\right)\right)\\&+\frac{1}{\pi^2}\left(0{\cdot}246\left(\frac{G^2}{4\pi}\right)^2-0{\cdot}07\left(\left(\frac{G_3{}^2}{4\pi}\right)^2+\left(\frac{G_4{}^2}{4\pi}\right)^2\right)-0{\cdot}153\left(\frac{G^2}{4\pi}\right)\left(\frac{G_3{}^2}{4\pi}\right)\right.\\&\left.-0{\cdot}133\left(\frac{G^2}{4\pi}\right)\left(\frac{G_4{}^2}{4\pi}\right)-0{\cdot}140\left(\frac{G_3{}^2}{4\pi}\right)\left(\frac{G_4{}^2}{4\pi}\right)\right)\quad . \quad . \quad . \quad (2.11\,a)\end{aligned}$$

$$\begin{aligned}\mu_{\mathrm{N}}=&\frac{1}{\pi}\left(-0{\cdot}822\,\frac{G^2}{4\pi}\right)+\frac{1}{\pi^2}\left(-0{\cdot}266\left(\frac{G^2}{4\pi}\right)^2+0{\cdot}09\left(\frac{G^2}{4\pi}\right)\left(\frac{G_3{}^2}{4\pi}\right)\right.\\&\left.+0{\cdot}364\left(\frac{G^2}{4\pi}\right)\left(\frac{G_4{}^2}{4\pi}\right)\right).\quad . \quad . \quad . \quad (2.11\,b)\end{aligned}$$

It is seen that up to second order no values of the coupling constants $G^2/4\pi$, $G_3{}^2/4\pi$, $G_4{}^2/4\pi$ can give both the proton and neutron magnetic moments correctly. For the symmetrical theory with

$$\frac{G^2}{4\pi}=\frac{G_3{}^2}{4\pi}=10,\qquad \frac{G_4{}^2}{4\pi}=0,$$

we get up to second order

$$\mu_{\mathrm{P}}=1{\cdot}35,\qquad \mu_{\mathrm{N}}=-2{\cdot}6.$$

Up to fourth order for the symmetrical theory the equations (2.11) give for

$$\frac{G^2}{4\pi}=\frac{G_3{}^2}{4\pi}=10$$

$$\mu_{\mathrm{P}}=4{\cdot}64,\qquad \mu_{\mathrm{N}}=-4{\cdot}36$$

and for the symmetrical theory, up to fourth order, no coupling constant $G^2/4\pi$ gives the right magnetic moments for both the proton and the neutron. But by a suitable addition of τ_4 coupling to the symmetrical theory—that is by taking $G_3=G$ and a suitable value of G_4—agreement with the observed values can be reached. But in this case the fourth order addition to the second order moment is large, and there is no reason to think that the sixth will not give an additional large correction.

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* These results are taken from the text of Nakabayasi and Sato's (1951) letter. There appears to be a discrepancy between the results in the text, and the fourth order symmetrical theory neutron magnetic moment given in table 1 of these authors.

And, in general, we have seen that the second order term with ps–ps coupling cannot give the right magnetic moments and indeed needs a large correction to reach the right magnetic moments. If the fourth order term supplies this, then it must be large, and so there is no reason to think that the sixth order term will not be large too. Thus until calculations are carried to a higher order on the coupling constant, we cannot simultaneously get the right answer from the terms in the series calculated, and the likelihood that all the succeeding terms are small.

The magnetic moment for the ps–pv case can only be worked out to second order, because of divergence difficulties in higher orders. From (2.11) the equivalence theorem (Case 1949 b) gives for the ps–pv case

$$\mu_P = 1 + \frac{1}{\pi}\left(\frac{2\kappa}{\mu}\right)^2\left(0{\cdot}348\,\frac{F^2}{4\pi} - 0{\cdot}237\left(\frac{F_3^2}{4\pi} + \frac{F_4^2}{4\pi}\right)\right), \quad . \quad (2.12\,a)$$

$$\mu_N = \frac{1}{\pi}\left(\frac{2\kappa}{\mu}\right)^2\left(-0{\cdot}822\,\frac{F^2}{4\pi}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.12\,b)$$

These are of exactly the same form as in the ps–ps coupling (the only difference being that the coupling constant is multiplied by $2\kappa/\mu$) and so the correct proton and neutron magnetic moments cannot simultaneously be obtained.

For scalar mesons with scalar coupling the magnetic moments up to second order are given by (Case 1949 a)

$$\mu_P = 1 + \frac{1}{\pi}\left(-1{\cdot}68\,\frac{g^2}{4\pi} + 0{\cdot}46\left(\frac{g_3^2}{4\pi} + \frac{g_4^2}{4\pi}\right)\right),$$

$$\mu_N = \frac{1}{\pi}\left(2{\cdot}6\,\frac{g^2}{4\pi}\right):$$

While, from the equivalence theorem scalar mesons with vector coupling give

$$\mu_P = 1, \qquad \mu_N = 0.$$

This is to say, the s–v coupling gives no anomalous magnetic moments to second order. Again, no adjustment of the coupling constants will give the right magnetic moments either from scalar or vector coupling.

The position in the perturbation theory treatment of magnetic moments is thus as follows. None of the three couplings (s–s, s–v, ps–pv) in which the magnetic moments have only been worked out to second order can give the correct magnetic moments. For two of these (s–v, ps–pv) the perturbation theory method cannot be pursued to higher orders because of divergence difficulties. The ps–ps coupling theory has been worked out to fourth order, and agreement with experiment can be obtained if certain particular values of G, G_3 and G_4 are chosen. But with this coupling (as well as with the other three couplings) there is no indication that for these particular values of the coupling constants sixth and higher order terms are small. Thus a final judgment on the value of perturbation theory cannot be given until more terms in the perturbation series are worked out, in order to see whether the series is going to converge fairly rapidly for the magnetic moment evaluation. With improvement of technique (Chisholm 1952) it may soon become possible to calculate sixth or even higher order terms and thus to get a picture of the behaviour of the perturbation power series.

(*b*) The strong coupling theory (Wentzel 1940, 1941, Pauli and Dancoff 1942, Dancoff and Serber 1943) gives the magnetic moment in terms of easily evaluated functions of the coupling constant, and thus does away with the fault of the perturbation approximations. The nucleon is considered as an infinitely heavy stationary source of mesons of finite extent (this picture then leaves out all processes involving nucleon pair creation). The finite nucleon model incidentally sets an upper limit to the momenta of the virtual mesons in the self-field of the nucleon, and thus forcibly eliminates high frequency divergences. The very forcefulness of this elimination may throw some doubt on the correctness of the strong coupling method, but, if the model is accepted s–s, s–v, ps–ps and ps–pv couplings can all be treated. However, as with both the pseudoscalar and the vector coupling nucleon pairs are likely to be important, it is unprofitable to consider these theories on the strong coupling theory of Wentzel.

It is found that the strong coupling theory gives the proton and neutron magnetic moments (Pauli and Dancoff 1942) as equal (except for a theory not involving charged mesons). This holds for all four types of coupling (though, as mentioned above, it is unprofitable to consider two of these). The result is directly contrary to experiment. In view of the drastic simplifying assumptions made in the strong coupling method this result probably cannot be taken as invalidating the ps–pv or s–s couplings.

2.2

The neutron possesses an intrinsic magnetic moment of $-1{\cdot}91$ nuclear magnetons in the direction of its spin. This will act upon the magnetic moment of the electron giving rise to a neutron–electron interaction. However, this direct magnetic dipole force is not usually called the neutron–electron interaction. The latter term is generally reserved for a short range ($\sim 10^{-13}$ cm) spin-independent interaction between a neutron and an electron which is given an electromagnetic interpretation by considering it to be a direct interaction of the neutron with the charge density responsible for an electromagnetic field. The experimental determinations of this interaction have been performed only where the charge density is that associated with electrons in atoms, hence the name neutron–electron interaction. Its magnitude is usually specified by the convention of giving the magnitude of the potential which must extent over a spherical volume of radius equal to the classical electron radius ($=2{\cdot}8\times 10^{-13}$ cm) which will give the same scattering matrix element at low energies as does the actual interaction. The present most accurate determination of this potential, which is in agreement with the results of previous investigations (for these see Foldy 1952 b) is that of Hughes, Harvey and Goldberg (1952). Their result for the potential is 4280±400 ev. It is this potential which meson field theory has to explain.

Let us suppose for the moment that the only electromagnetic effect of the neutron's coupling with the meson field is the anomalous magnetic moment, μ_N Bohr nuclear magnetons, of the neutron. Then the neutron in an electromagnetic field can be described by the Dirac equation with the Pauli (1941) moment term:

$$\gamma_\mu \frac{\partial \psi}{\partial x_\mu} + \kappa\psi - \tfrac{1}{2} i \mu_N \left(\frac{e}{2\kappa}\right) \gamma_\mu \gamma_\nu \psi \left(\frac{\partial A_\mu}{\partial x_\nu} - \frac{\partial A_\nu}{\partial x_\mu}\right) = 0. \quad . \quad . \quad (2.21)$$

This equation may be written

$$i\frac{\partial \psi}{\partial t} = \left\{\beta\kappa + \boldsymbol{\alpha} \,.\, \boldsymbol{p} - \left(\frac{\mu_N e}{2\kappa}\right)\beta \boldsymbol{H} \,.\, \boldsymbol{\sigma} + i\beta\boldsymbol{\alpha} \,.\, \boldsymbol{E}\left(\frac{\mu_N e}{2\kappa}\right)\right\}\psi. \quad . \quad (2.22)$$

Here $F_{\nu\mu} = \dfrac{\partial A_\mu}{\partial x_\nu} - \dfrac{\partial A}{\partial x_\mu} = (\boldsymbol{H},\ i\boldsymbol{E})$ is the external electromagnetic field.

$\boldsymbol{p} = p_j = -i\dfrac{\partial}{\partial x_j}$ is the momentum operator.

Foldy (1952 a, b) suggests that the physical results of the Pauli moment term in these equations may be made more perspicuous by transforming the equation (2.22) into a non-relativistic form. To do this he suggests the canonical transformation

$$\psi' = e^{iS_2} e^{iS_1} \psi, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.23)$$

where

$$S_1 = -\frac{i}{2\kappa}\beta\left(\boldsymbol{p} + i\beta\boldsymbol{E}\frac{\mu_N e}{2\kappa}\right) . \,\boldsymbol{\alpha}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.24)$$

$$S_2 = -\frac{i}{2\kappa}\beta\left(-\frac{\mu_N e}{(2\kappa)^2}\gamma_5\,(\boldsymbol{H} \,.\, \boldsymbol{p}) - (\boldsymbol{p} \,.\, \boldsymbol{\alpha})\frac{p^2}{2\kappa^2}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.25)$$

Then, up to and including the second order in $1/\kappa$ (omitting a dipole term),

$$i\frac{\partial \psi'}{\partial t} = \left\{\left(\beta\kappa + \frac{\beta}{2\kappa}p^2\right) - \frac{\mu_N e}{(2\kappa)^2}\,\mathrm{div}\,\boldsymbol{E} - \frac{\mu_N e}{2\kappa}\boldsymbol{\sigma} \,.\, \boldsymbol{H} - \frac{\mu_N e}{2\kappa^2}\boldsymbol{\sigma} \,.\, (\boldsymbol{E} \times \boldsymbol{p})\right\}\psi'. \quad . \quad . \quad . \quad (2.26)$$

If μ_N were zero this would just be the usual non-relativistic Hamiltonian. The third term on the right-hand side of (2.26) is a magnetic interaction and does not concern us at the moment. The fourth term is a spin orbit coupling, and the neutron–electron interaction that is being looked for is essentially spin independent. The second term on the right-hand side of (2.26) provides a spin-independent short-range interaction with the Coulomb field of the electron (Foldy 1951). If $\boldsymbol{E}$ is the Coulomb field of the electron situated at the point $\boldsymbol{x}_e$ and $\boldsymbol{x}$ the neutron coordinate the equation (2.26) becomes

$$i\frac{\partial\psi'}{\partial t}=\left\{\left(\beta\kappa+\frac{\beta}{2\kappa}p^2\right)+\frac{4\pi\,\mu_{\mathrm{N}}e^2}{(2\kappa)^2}\delta(\boldsymbol{x}-\boldsymbol{x}_e)\right.$$
$$\left.-\frac{\mu_{\mathrm{N}}e}{2\kappa}\boldsymbol{\sigma}\,.\,\boldsymbol{H}-\frac{\mu_{\mathrm{N}}}{2\kappa^2}\boldsymbol{\sigma}\,.\,(\boldsymbol{E}\times\boldsymbol{p})\right\}\psi' \quad . \quad . \quad . \quad (2.27)$$

and evaluation shows that the second term in (2.27) gives a neutron–electron interaction of 4080 ev (Foldy 1951, 1952 b). This would explain nearly the whole of the neutron–electron interaction, as it leaves an interaction of the order at most of 500 ev and probably less.

The electron and neutron satisfy the condition of the Foldy transformation of moving at non-relativistic relative velocities. This explicit exhibition of the inherent properties of the Pauli moment interaction by transforming to a non-relativistic equation, is entirely comparable to the similar exhibition of the properties of the Dirac interaction ($-ieA_\mu\gamma_\mu$) of an electron. In the latter case it is the Dirac magnetic moment of one Bohr magneton that becomes manifest.

It can now be seen that, given the Foldy derivation of the major part of the neutron–electron interaction, a problem is posed in meson field theory. For, entirely separate from the magnetic moment, from which the Foldy interaction arises, the coupling of the neutron to the meson field would be expected to give rise to the following effect. The neutron will disassociate itself into a nucleon along with various π-mesons according to the scheme

$$\begin{aligned}\mathrm{n}&\rightarrow \mathrm{p}+\pi^-\\&\rightarrow \mathrm{n}+\pi^++\pi^-\\&\rightarrow \mathrm{p}+\pi^++2\pi^-\end{aligned}$$

where n, p, stand for neutron and proton and π^-, π^+ for negatively and positively charged π-mesons. In the ' state ' $\mathrm{p}+\pi^-$ the proton and meson will not be in the same space configuration, so there will be a distribution of charge within a region of the order of the meson Compton wavelength. (A similar effect should arise from virtual pair creation processes.) Thus when the electron approaches within this distance of the neutron, a neutron–electron interaction should result. Now it has been shown that this effect is only required to dispose of, at most, about 500 ev of the electron–neutron interaction. But when this ' charge-distribution ' interaction is worked out in first order perturbation in meson theory, an interaction an order of magnitude larger than 800 ev is obtained.

The discrepancy is shown in table 1. Let V' be that part of the neutron–electron interaction due to the ' charge distribution ' effect. Then $4\pi/3\,(e^2/m)^3V'$ is its volume integral. If μ_{N} is the number of nuclear magnetons in the neutron magnetic moment, then $(e/2\kappa)^2\mu_{\mathrm{N}}$ is of the same dimensions as $4\pi/3\,(e^2/m)^3V'$ (and is incidentally the volume integral of the Foldy part of the neutron–electron interaction). Then V' and μ_{N} can be worked out by perturbation theory to second order in the meson–nucleon coupling constant (for μ_{N} see § 2.1 and for V' see Luttinger

1948, Slotnick and Heitler 1949, Case 1949 a, Duncoff and Drell 1949, Berowitz and Kohn 1949). The ratio $4\pi/3\,(e^2/m)^3 V'/(e/2\kappa)^2\,\mu_N$ is given in table 1 as worked out in various meson theories. The experimental ratio (with $V'=-70\pm400$ ev and $\mu_N=-1{\cdot}93$) is also given.

The equivalence theorem shows, that up to second order, the charge distribution neutron–electron interaction is the same in ps–pv coupling as in ps–ps except for a factor $(2\kappa/\mu)^2$. And, also to the second order, that there is no charge distribution neutron–electron interaction in the s–v coupling.

Table 1

Source	$\frac{4\pi}{3}\left(\frac{e^2}{m}\right)^3 V' \Big/ \left(\frac{e}{2k}\right)^2 \mu_N$
Symmetrical ps–ps	0·318
Charged ps–ps	0·318
Symmetrical s–s	−9·05
Charged s–s	−9·05
Experimental	0·02 ±0·10

Thus the ‘ charge-distribution ’ part of the neutron–electron interaction, as worked out in second order meson perturbation theory, has very much too large a value. As in the magnetic moment case, fourth and higher order terms could be worked out for the ps–ps and s–s couplings, while non-renormalizable infinities are a bar to such calculations in the other two couplings.

§ 3. THE TWO-NUCLEON SYSTEM

In this chapter an account will be given of the attempts made in π-meson field theory to explain the experimental facts on nucleon–nucleon forces. In § 3.1 the experimental data will be given, and in the succeeding §§ 3.2–3.6 the theoretical attempts at explanation will be set down.

3.1

The experimental data may be listed under the following headings :—

(*a*) *The Deuteron*

$$\left.\begin{array}{ll}\text{Binding energy} & : (2{\cdot}227\pm0{\cdot}003)\ \text{Mev.}\\ \text{Electric quadrupole moment} & : (2{\cdot}738\pm0{\cdot}016)\ 10^{-27}\ \text{cm}^2.\\ \text{Magnetic moment} & : (0{\cdot}8568\pm0{\cdot}0004)\ \text{nuclear magnetons.}\end{array}\right\} \quad (3.11)$$

As the deuteron is in a $^3S+^3D$ state, the magnetic moment depends directly on the proportion of D state. If p_D is the proportion of D state, the non-relativistic expression for the deuteron magnetic moment $\mu_D=\mu_N+\mu_P-3/2\,(\mu_N+\mu_P-\frac{1}{2})\,p_D$ leads to $p_D=0{\cdot}04$. The relativistic corrections are, however, usually estimated at about 2%, leading to $0{\cdot}02\lesssim p_D\lesssim0{\cdot}06$.

(*b*) *Proton–neutron Scattering*

In the energy region from 0 to about 10 MeV S-wave scattering alone is of importance. General arguments (Schwinger 1947, Bethe 1949, Blatt and Jackson 1949) show that for reasonable shapes (including all the commonly used ones) of the nuclear potential the phase shift δ is given in terms of the centre of mass coordinates wave number k, by

$$k \cot \delta = -\frac{1}{a} + \tfrac{1}{2} r_0 k^2 + O\,(k^4 r_0^3). \quad . \quad . \quad . \quad . \quad (3.12)$$

Here a and r_0 are constants, a is the scattering length of Fermi and r_0 is the effective range. The term $O(k^4 r_0^3)$ is negligible for energies below about 10 MeV.

Experiment confirms (Blatt and Jackson 1949) this formula for the phase shift, and determines the constants a and r_0. The phase shifts are different in the singlet and triplet states, and serve to determine different scattering lengths and effective ranges.

The triplet scattering length is related to the deuteron binding energy, ϵ, and the triplet effective range by

$$-\frac{1}{a} = \sqrt{\kappa\epsilon} + \tfrac{1}{2}\kappa\epsilon r_0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.13)$$

Thus the low energy proton–neutron scattering essentially determines three fresh parameters (Salpeter 1951):

$$\left.\begin{array}{ll} \text{Triplet effective range } {}^3r_0 & =(1{\cdot}70 \pm 0{\cdot}030)\ 10^{-13}\ \text{cm.} \\ \text{Singlet effective range } {}^1r_0 & =(2{\cdot}7 \pm 0{\cdot}5)\ 10^{-13}\ \text{cm.} \\ \text{Singlet scattering length } a_s & =(-23{\cdot}68 \pm 0{\cdot}06)\ 10^{13}\ \text{cm.} \end{array}\right\} \quad . \quad (3.14)$$

In the energy region from about 15 to about 25 MeV the S-wave phase shift is nearly 90°. Thus S-wave scattering (with total cross-section $\sim \sin^2 \delta$) dominates higher angular momentum scattering, which cannot be effectively observed.

For energies greater than about 25 MeV higher angular momenta enter significantly. The differential scattering cross-sections at 40, 90 and 260 MeV are given in fig. 3 (Kelly *et al.* 1950).

(*c*) *Proton–proton Scattering*

Similarly to the proton–neutron scattering, the proton–proton scattering S-wave phase shift for energies below about 10 MeV, is given theoretically by

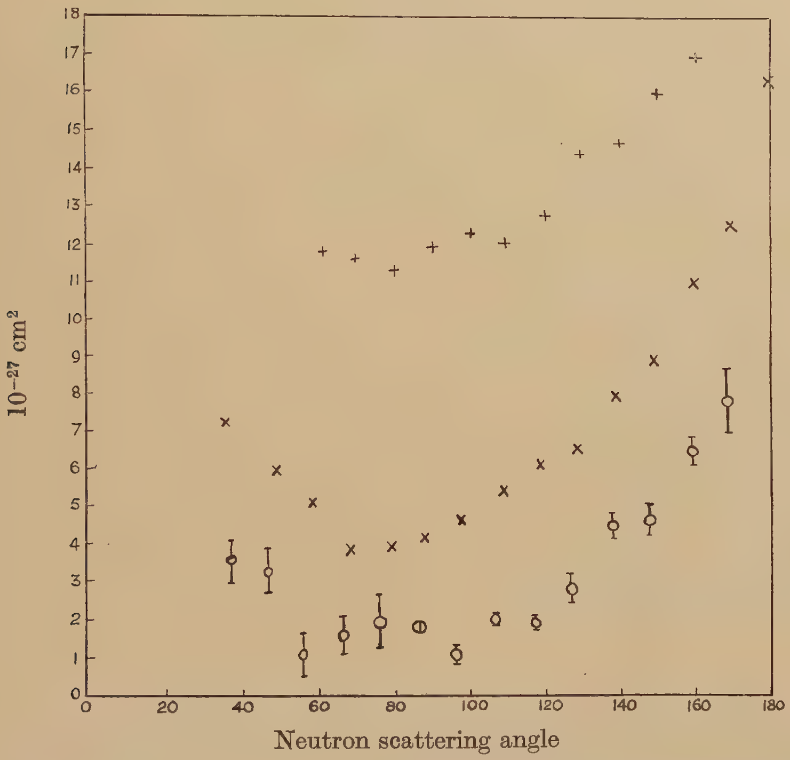
$$g(\eta) + \frac{2\pi \cot \delta}{e^{2\pi\eta} - 1} = D\left(-\frac{1}{a'} + \tfrac{1}{2} r_0{}' k^2\right) \quad . \quad . \quad . \quad . \quad (3.15)$$

where $$D = \frac{2}{\kappa e^2}\ (=5{\cdot}76 \times 10^{-12}\ \text{cm}) \qquad \eta = \frac{1}{kD}$$

and $$g(\eta) = -2 \ln \eta + 2\eta^2 \sum_{\nu=1}^{\infty} \nu^{-1} (\nu^2 + \eta^2)^{-1}.$$

The difference in form between (3.15) and (3.12) is due to the Coulomb scattering. Again it is found that the experimental points fit very well the law given by (3.15). And the experiments show that if the proton–proton nuclear potential (in the singlet S-state) is assumed to have the same range and shape as the neutron–proton force (in the singlet S-state) then the depth only differs by 1·5–3%, for reasonable potential well shapes (Bethe 1949, Breit *et al.* 1939). Schwinger (1950) has shown that this difference can be accounted for by the different magnetic dipole interactions of the proton–proton and proton–neutron system. Thus the data below 10 Mev is in conformity with the charge independence hypothesis. Unfortunately, the scattering below 10 Mev provides no information on the relative shape and range of the neutron–proton and proton–proton wells.

Fig. 3



Differential neutron–proton cross-section in the centre-of-mass system in 10^{-27} cm^2 per steradian. ○ : 260 Mev, × : 90 Mev, + : 40 Mev.

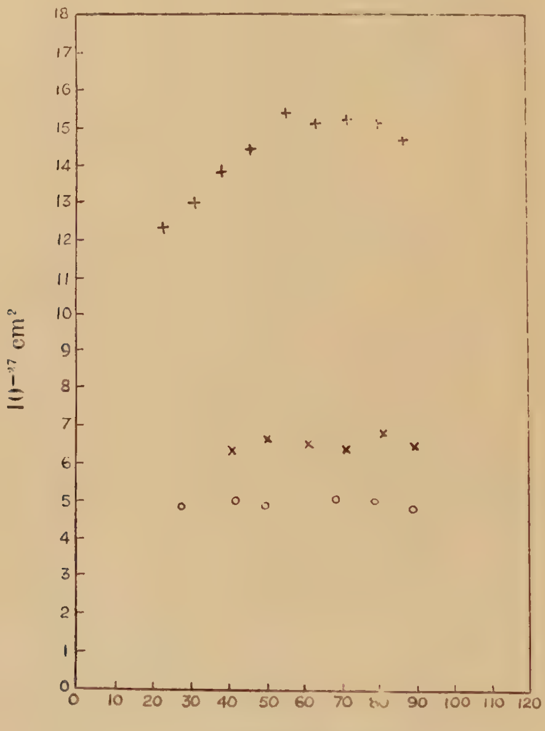
For the same reason as in the neutron–proton case, the experiments from about 15 Mev to about 25 Mev provide little information. Scattering experiments have been performed at 30 Mev (Panofsky and Fillmore 1950, Cork *et al.* 1950), 75 Mev, 100 Mev (Birge *et al.* 1951), 240 Mev (Oxley and Schamberger 1952) and 348 Mev (Chamberlain *et al.* 1951). Some of these results are given in fig. 4.

The results of high energy nucleon–nucleon scattering experiments have been given for completeness. In fact, almost no serious attempt has been made to describe high energy scattering on meson theory.

3.2

The most usual method of evaluating the motions of nucleons in meson theory, is to find, by some approximation method, the potential due to meson exchange and then substitute this potential in a Schrödinger or other equation of motion for the two particles. The virtual mesons do not appear explicitly in this equation. The perturbation expansion in a power series of the meson–nucleon coupling constant is a method of approximation ready to hand. It is used despite the fact that it is doubtful (as mentioned before in connection with the single nucleon problem) whether the few terms of the series that can be worked out represent a good approximation to its sum, in view of the large nucleon–meson coupling constant.

Fig. 4



Scattering Angle

Differential proton–proton cross-section in the centre-of-mass system in 10^{-27} cm^2 per steradian ○ : 240 Mev, × : 75 Mev, + : 32 Mev.

In this section the simplest attempts (Yukawa 1935, Kemmer 1938 a, Wentzel 1949) at a meson theory of nuclear forces are set down, using a second order potential. This implies the exchange of only one virtual meson. In deriving these potentials for the various meson theories (1.25 *a*, *b*, *A*, *B*) the static approximation is used. In this static approximation the change in energy of the nucleons due to the recoil of the emitted and absorbed virtual mesons is neglected. The potential at close distances of approach comes from the exchange of mesons of large momenta. Thus the static approximation potential certainly is not correct for distances of approach of the order of the nucleon Compton wave length. Thus the method is suitable, and has been applied to, low energy rather than high energy scattering.

The static second order potentials resulting from the Hamiltonians (1.25 *a*, *b*, *A*, *B*) are studied in the following paragraphs (*a*), (*b*), (*A*), (*B*) respectively. (Moller and Rosenfeld 1940, have also separated these potentials by a canonical transformation method.)

(*a*) *Scalar Mesons with Scalar Coupling*

The Hamiltonian for the interaction potential is found to be

$$\int d^3x_1\, d^3x_2\, \Psi^*(x_1)\, \Psi^*(x_2)\, \frac{-1}{4\pi} \sum_{\lambda=1}^{4}{}' \,(g^2{}_\lambda\, \tau_\lambda{}^{(1)}\tau_\lambda{}^{(2)})\, \beta^{(1)}\beta^{(2)}\, \frac{e^{-\mu r}}{r}\, \Psi(x_1)\, \Psi(x_2) \quad \ldots\ldots\ (3.21)$$

where $\beta^{(1)}\tau_\lambda{}^{(1)}$, $\beta^{(2)}\tau_\lambda{}^{(2)}$ operate on the particles at $\mathbf{x}_1$ and $\mathbf{x}_2$ respectively, and $r = |\mathbf{x}_1 - \mathbf{x}_2|$. More shortly we may say the potential is

$$-\frac{1}{4\pi} \sum_{\lambda=1}^{4}{}' \,(g_\lambda{}^2\tau_\lambda{}^{(1)}\tau_\lambda{}^{(2)}) \,.\, \beta^{(1)}\beta^{(2)}\, \frac{e^{-\mu r}}{r} . \quad \ldots\ldots\ (3.22\,a)$$

The constants g_λ take on various values according to what kind of coupling we assume. However, it can be seen that for any values of g_λ this is an inadequate potential for the description of proton–neutron forces. For it is a non-spin dependent potential, and thus the only difference in binding energy of the triplet and singlet states in a bound state of the proton–neutron system is that due to the magnetic interaction of the proton and neutron. According to Casimir (1936) this difference is $\lesssim 0{\cdot}1$ Mev. As the singlet bound state is virtual, the actual energy difference between triplet and singlet states is of the order of the deuteron binding energy. Thus the magnetic interaction alone cannot explain the triplet–singlet splitting. For this reason it is justifiable to reject the potential (3.22 *a*).

(*b*) *Scalar Mesons with Vector Coupling*

Case (1949 b) in his proof of the equivalence theorem incidentally shows that a contact transformation eliminates the interaction up to and including terms of second order in *f*. Thus to that order this theory gives no potential.

(*A*), (*B*). *Pseudoscalar Mesons with Pseudoscalar and Pseudovector Coupling*

The equivalence theorem shows that to second order in the coupling constant these two theories give the same nucleon–nucleon interaction. However, if the nucleon–nucleon potential is calculated in the static approximation, it reveals itself in different forms for the two theories.

For pseudoscalar mesons with pseudoscalar coupling the potential is

$$\frac{1}{4\pi} \sum_{\lambda=1}^{4} (G_\lambda{}^2\tau_\lambda{}^{(1)}\tau_\lambda{}^{(2)})\, \beta^{(1)}\, \gamma_5{}^{(1)}\, \beta^{(2)}\, \gamma_5{}^{(2)}\, \frac{e^{-\mu r}}{r} \quad \ldots\ (3.22\,A)$$

and for pseudoscalar mesons with pseudovector coupling, in the non-relativistic approximation

$$\frac{1}{4\pi}\sum_{\lambda=1}^{4}(F_\lambda^2\tau_\lambda^{(1)}\tau_\lambda^{(2)})\left(\tfrac{1}{3}\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}+\left(1+\frac{3}{\mu r}+\frac{3}{\mu^2r^2}\right)\right.$$
$$\left.\times\left(\frac{(\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{r})(\boldsymbol{\sigma}^{(2)}\cdot\boldsymbol{r})}{r^2}-\tfrac{1}{3}\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}\right)\right)\frac{e^{-\mu r}}{r}. \quad . \quad . \quad . \qquad (3.22\,B)$$

It can be shown directly that in the non-relativistic region (that is for distances of approach greater than the nucleon Compton wavelength) where the derivation of (3.22 *B*) holds, that (3.22 *A*) is of the form (3.22 *B*). In fact, in the non-relativistic region (3.22 *A*) reduces to (Araki 1949)

$$\frac{1}{4\pi}\sum_{\lambda=1}^{4}\left(\frac{\mu}{2\kappa}\right)^2(G_\lambda^2\tau_\lambda^{(1)}\tau_\lambda^{(2)})\left(\tfrac{1}{3}\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}+\left(1+\frac{3}{\mu r}+\frac{3}{\mu^2r^2}\right)\right.$$
$$\left.\times\left(\frac{(\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{r})(\boldsymbol{\sigma}^{(2)}\cdot\boldsymbol{r})}{r^2}-\tfrac{1}{3}\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}\right)\right)\frac{e^{-\mu r}}{r}. \quad . \quad . \quad . \qquad (3.23)$$

As the derivation of the potential (3.23), (3.22 *B*) only holds in the non-relativistic region, it does not follow that the nucleon–nucleon potential has an r^{-3} singularity at small distances. In fact Van Hove (1949) has found by taking into account the non-static part of the interaction that the singularity at the origin is of order only r^{-2}.

In addition to being wrong, the potentials (3.22 *A*, *B*) exert a great influence on the system, at small (relativistic) distances. In fact, the singularity makes it impossible to solve the bound state problem. Thus at distances of the order of half the π-meson Compton wavelength the potential (3.22 *B*) (3.23) must be replaced by some other expression. Ferreti (1941) showed that with a cut-off the cut-off radius had to be chosen unreasonably large (of the order of the meson Compton wavelength) in order to give the right singlet–triplet energy splitting for the deuteron. Taketani *et al.* (1951 a) discuss the reasons for replacing the π-meson potential at relativistic distances by a phenomenological potential. The reasons include, besides the importance of recoil at small distances, the possible influence of isobars and heavier mesons, and also the invalidity (Levy 1952 b) of the perturbation expansion at small distances.

These authors (Taketani *et al.* 1951 b, Machida *et al.* 1951) discuss the two-body problem with ps–pv coupling of π-mesons. A phenomenological potential at small distances, and the π-meson potential up to fourth order at others, is used. Levy (1952 b) has investigated the effect of higher order perturbations on the nucleon–nucleon potential for pseudoscalar mesons with pseudoscalar coupling. These more refined treatments of the couplings (1.25 *A*), (1.25 *B*) (that is the couplings with pseudoscalar mesons) are reported in the following § 3.3.

3.3

Levy (1952 b) has recently made an attempt at finding the effect of higher order terms in the perturbation series for the potential, using pseudoscalar mesons with pseudoscalar symmetric coupling. He has avoided the difficulties of extreme relativistic velocities of the proton and neutron (and the possible effect of heavy mesons) by imposing the condition that the wave function should be zero for the distance between the proton and the neutron less than a certain small critical distance r_c. r_c is left as a parameter to be determined by the fitting of experimental results. This assumption that the wave function tends to zero at a small critical distance r_c is equivalent to the assumption of an infinite repulsive potential at small distances (which is just the assumption that Jastrow (1951) uses for spin singlet states to explain high energy proton–neutron and proton–proton scattering on the symmetric theory).

With this assumption, and using the Tamm–Dancoff non-adiabatic method (Dancoff 1950, Levy 1952 a, see also § **3.4** below), Levy effectively makes a term by term analysis of the potential perturbation series in the coupling constant. It is found that the nucleon–nucleon interaction, outside the critical radius r_c, can be represented by an effective potential

$$\begin{aligned}V(\boldsymbol{r}) = \boldsymbol{\tau}^{(1)} \cdot \boldsymbol{\tau}^{(2)} \frac{G^2}{4\pi}\left(\frac{\mu}{2\kappa}\right)^2 \Bigg(& \tfrac{1}{3}\boldsymbol{\sigma}^{(1)} \cdot \boldsymbol{\sigma}^{(2)} \\ & + \left(1+\frac{3}{\mu r}+\frac{3}{\mu^2 r^2}\right)\left(\frac{(\boldsymbol{\sigma}^{(1)} \cdot \boldsymbol{r})(\boldsymbol{\sigma}^{(2)} \cdot \boldsymbol{r})}{r^2} - \tfrac{1}{3}\boldsymbol{\sigma}^{(1)} \cdot \boldsymbol{\sigma}^{(2)}\right)\Bigg)\frac{e^{-\mu_r}}{r} \\ & - 3\left(\frac{G'^2}{4\pi}\right)^2\left(\frac{\mu}{2\kappa}\right)^2\frac{1}{\mu r^2}\left(\frac{2}{\pi}K_1(2\mu r)+\frac{\mu}{2\kappa}\left(\frac{2}{\pi}K_1(\mu r)\right)^2\right) \quad . \quad . \quad . \quad (3.31)\end{aligned}$$

where $K_1(x)$ is the first order Hankel function of imaginary argument. Consequently, the last term of (3.31) is for large r asymptotically equal to

$$-3\left(\frac{G'^2}{4\pi}\right)^2\left(\frac{\mu}{2\kappa}\right)^2\frac{1}{\mu r^2}\left(1+\frac{\mu}{\kappa}(\pi\mu r)^{-1/2}\right)(\pi\mu r)^{-1/2}\, e^{-2\mu r}. \quad . \quad (3.32)$$

$G'^2/4\pi$ is a ‘ renormalized ’ coupling constant whose value (though ideally given in terms of G, μ, κ by the theory) is mathematically very difficult, if not impossible, by present methods, to obtain $\boldsymbol{\tau}^{(1)}\boldsymbol{\tau}^{(2)} = \sum_{j=1}^{1} \tau_j^{(1)}\tau_j^{(2)}$. Some terms of higher than second order in $G^2/4\pi$ are incorporated in the $(G'^2/4\pi)^2$ term. Levy shows that other higher order terms provide only small corrections to the inter-nucleon force in the non-relativistic region. Qualitative arguments are given to indicate that the π-meson potential probably becomes repulsive in the relativistic ($r < r_c$) region. These latter arguments lend some measure of support to the hypothesis of a total infinite repulsive potential at small distances due to π-mesons and also to heavier mesons and isobaric states of nucleons, and hence lend that measure of support to the hypothesis that the wave function tends to zero at a critical distance.

In the potential $V(\boldsymbol{r})$ ($r \geqslant r_c$) Levy puts $G' = G$. Then $G^2/4\pi$ and r_c are determined by fitting exactly the binding energy of the deuteron and the proton–neutron singlet scattering length (3.11, 3.14). It is found that

$$r_c = (0{\cdot}38 \pm 0{\cdot}03)\frac{1}{\mu} = (0{\cdot}53 \pm 0{\cdot}04)\ 10^{-13}\ \text{cm}$$

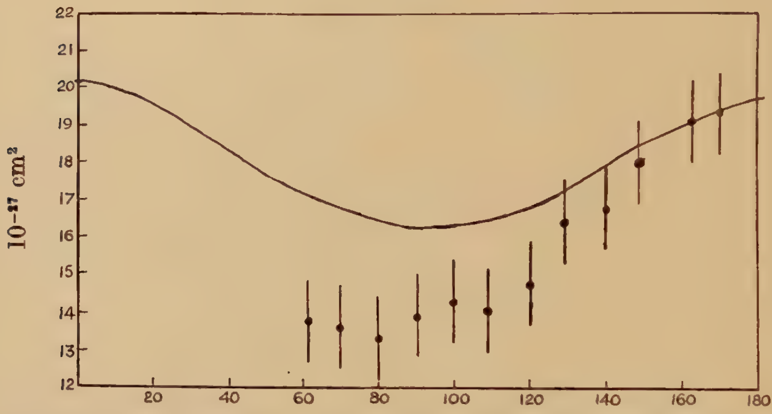
$$G^2/4\pi = 9{\cdot}7 \pm 1{\cdot}3\ .$$

From these the other significant constants for the proton–neutron system can be derived with the results shown in table 2.

Table 2

	Calculated	Observed
Singlet effective range	$2{\cdot}5\times10^{-13}$ cm	$(2{\cdot}7\pm0{\cdot}5)\times10^{-13}$ cm
Triplet effective range	$1{\cdot}66\times10^{-13}$ cm	$(1{\cdot}704\pm0{\cdot}03)\times10^{-13}$ cm
Electric quadrupole moment	$2{\cdot}08\times10^{-27}$ cm^2	$(2{\cdot}738\pm0{\cdot}016)\times10^{-27}$ cm^2
Proportion of D-state	$p_D=0{\cdot}05$	$0{\cdot}02\lesssim p_D\lesssim0{\cdot}06$

Fig. 5



Neutron–proton differential scattering cross-section in the centre-of-mass system at 40 MeV. The theoretical curve is that of Levy. The experimental points are here normalized to make the total cross-section agree with the mean experimental value, and an arbitrary experimental error of 10% is assumed.

Calculation of the neutron–proton scattering at 40 MeV gives the curve shown in fig. 5, where the experimental points are also given for reference.

A comparison with experiment when G' is put equal to G is thus quite good (except for the quadrupole moment). However, the position remains unsatisfactory to some extent until G' is determined in terms of G, and it is checked that G' does not have such a value as to make agreement with experiment impossible for reasonable values of G and r_0. As the potential (3.31) is charge independent, and gives the correct low energy neutron–proton scattering, it automatically gives the right low energy proton–proton scattering.

Taketani, Machidi and Onuma (Taketani *et al.* 1951 b, Machida *et al.* 1951) give the following expression for the static π-meson symmetric theory ps–pv ((1.25 B) $F_1=F_2=F_3$, $F_4=0$) potential up to fourth order in the coupling constant.

$$\frac{F^2}{4\pi}\boldsymbol{\tau}^{(1)}\boldsymbol{\tau}^{(2)}\left(\tfrac{1}{3}\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}+\left(1+\frac{3}{\mu r}+\frac{3}{\mu^2r^2}\right)\left(\frac{(\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{r})(\boldsymbol{\sigma}^{(2)}\cdot\boldsymbol{r})}{r^2}-\tfrac{1}{3}\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}\right)\right)$$
$$\times\frac{e^{-\mu r}}{r}+\left(\frac{F^2}{4\pi}\right)^2\mu\left(\boldsymbol{\tau}^{(1)}\cdot\boldsymbol{\tau}^{(2)}\,U_\tau(\mu r)+\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}\,U_\sigma(\mu r)\right.$$
$$\left.+3\left(\frac{(\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{r})(\boldsymbol{\sigma}^{(2)}\cdot\boldsymbol{r})}{r^2}-\tfrac{1}{3}\,\boldsymbol{\sigma}^{(1)}\,\boldsymbol{\sigma}^{(2)}\right)U_T(\mu r)\right)\quad\ldots\ldots\quad(3.33)$$

where

$$U_\tau(x)=\frac{8}{\pi}\left(-\frac{K_0''''(2x)}{x}+\frac{2K_0'''(2x)}{x^2}-\frac{4K_0''(2x)}{x^3}+\frac{2K_0(2x)}{x^4}\right),$$
$$U_\sigma(x)=-\frac{8}{\pi}\left(\frac{2K_0'''(2x)}{x^2}-\frac{2K_0''(2x)}{x^3}+\frac{K_0'(2x)}{x^4}\right),$$
$$U_T(x)=\frac{4}{\pi}\left(\frac{2K_0'''(2x)}{x^2}-\frac{5K_0''(2x)}{x^3}+\frac{4K_0'(2x)}{x^4}\right),$$

where $K_0(x)$ is the zero order Hankel function of imaginary argument. The effect of higher order terms in the coupling constant is not inspected.

Table 3 (i). Zero Cut-Off

$F^2/4\pi$	${}^1r_0$	r_c
0·075	$1{\cdot}925\times10^{-13}$ cm	$0{\cdot}589\frac{1}{\mu}$
0·090	$2{\cdot}457\times10^{-13}$ cm	$0{\cdot}665\frac{1}{\mu}$

Table 3 (ii). Infinite Repulsive Square Well Inside r_c.

$F^2/4\pi$	$1\,{}^1r_0$	r_c
0·075	$2{\cdot}104\times10^{-13}$ cm	$0{\cdot}329\frac{1}{\mu}$
0·090	$2{\cdot}585\times10^{-13}$ cm	$0{\cdot}384\frac{1}{\mu}$

The authors investigate the low energy properties of the proton–neutron system using the static potential (3.33) outside a critical distance, r_c, and a phenomenological potential inside. Two kinds of phenomenological potential are considered. (i) A zero potential inside (the zero cut-off). (ii) An infinite repulsive potential inside (equivalent to the assumption of Levy that the wave function tends to zero at the critical distance).

(1) Firstly the singlet neutron–proton scattering is considered. A particular coupling constant is taken and r_c is chosen to fit the singlet

scattering length. From the values of $F^2/4\pi$ and r_c, the singlet effective range is calculated with the results shown in tables 3 (i) and 3 (ii).

(2) Secondly the deuteron ground state, triplet neutron–proton scattering, quadrupole moment and the percentage of D state is considered. It is found that the infinite repulsive cutting off is unable to produce a sufficient deuteron binding energy. Consequently only the results for the zero cut-off are listed in table 4. Here, for a given $F^2/4\pi$, r_c is found to give the correct binding energy. Then the other quantities listed are found.

Table 4

$F^2/4\pi$	Q (cm²)	$^3r_0$ (cm)	P_D	r_c
0·05	$1{\cdot}85\times10^{-27}$	$1{\cdot}325\times10^{-13}$	0·074	$0{\cdot}16\ \frac{1}{\mu}$
0·08	$2{\cdot}37\times10^{-27}$	$1{\cdot}582\times10^{-13}$	0·086	$0{\cdot}232\frac{1}{\mu}$
0·10	$3{\cdot}25\times10^{-27}$	$1{\cdot}75\ \times10^{-13}$	0·113	$0{\cdot}303\frac{1}{\mu}$

The experimental values of these quantities are

$$^1r_0=(2{\cdot}7\pm0{\cdot}5)\times10^{-13}\text{ cm},\quad ^3r_0=(1{\cdot}7\pm0{\cdot}03)\times10^{-13}\text{ cm},$$
$$Q=(2{\cdot}738\pm0{\cdot}016)\ 10^{-27}\text{ cm}^2.$$

3.4

The last two sections have dealt with the efforts to describe the low-energy behaviour of the two-nucleon system by the adiabatic method. By the adiabatic method is here meant one in which a two-body potential is first found from π-meson theory and then the problem solved by treating this potential as the only manifestation of the π-meson field. Also the π-meson potentials that have been considered have been static, derived by neglect of the nucleon recoil, and thus their region of validity is essentially limited to non-relativistic distances. Some authors have made experimental attempts to break away from these limitations.

Araki (1951) discusses the nucleon–nucleon interaction to second order in the coupling constant with recoil taken into account for the s–s and ps–ps couplings. He does not only discuss the effective potential when the matrix element is taken between positive energy states, but also the case when some (or all) of these states belong to negative energies. Araki finds that the effect of negative energy states and recoil leads to an important change in the potential for ps–ps coupling, but no significant change in the scalar case.

The non-adiabatic method of Tamm (1945) and Dancoff (1950) is applied by these authors to neutral scalar mesons with scalar coupling interacting with two nucleons. It consists in writing down an approximate wave function for the system, consisting of a superposition of wave functions for two nucleons alone and wave functions for two nucleons and one meson. By eliminating the latter type from the equation for the wave function for the system, an eigenvalue equation for the binding energy is obtained. (The method may, ideally, be refined to any desired degree by taking into account two, three or more mesons.)

The results of the adiabatic approximation are regained when non-relativistic approximations are made in the eigenvalue equation. However, when relativistic terms are taken into account a spin–orbit coupling term appears in the eigenvalue equation.

Another non-adiabatic (in the above sense) treatment of the two body bound state problem is the integral equation method of Salpeter and Bethe (1951). The authors investigated the equation (since derived rigorously from field theory by Gell-Mann and Low (1951))

$$\psi(3, 4) = -i \int\int\int\int d\tau_5\, d\tau_6\, d\tau_7\, d\tau_8\, K_{+a}(3, 5)\, K_{+b}(4, 6)\, \bar{G}(5, 6;\ 7, 8)\psi(7, 8).$$

$\psi(3, 4)$ is the wave function for the particles a and b being at the space time point 3, 4 if they are at the space time point 7, 8 with the wave-function $\psi(7, 8)$. K_{+a}, K_{+b} are the Feynman (1949) propagation factors for the particles a and b. The kernel, $\bar{G}$, of the equation consists of a sum of terms corresponding to certain ‘ irreducible ’ transitions of the two Fermi particles. Salpeter and Bethe solve the integral equation by approximating $\bar{G}$ by the single term corresponding to the Feynman diagram shown in fig. 6.

Fig. 6

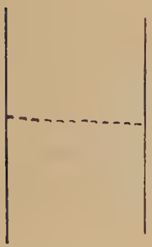


Fig. 7

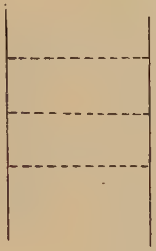
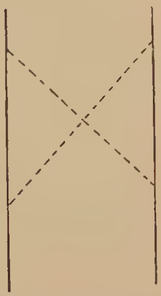


Fig. 8



The solution of the integral equation then includes those transitions of the system corresponding to fig. 7. These ladder type graphs come from an iteration of the kernel corresponding to fig. 6. On the other hand, the solution of the integral equation with the kernel corresponding to fig. 6 does not contain ‘ cross-over ’ terms corresponding to fig. 8. (Levy (1952 b) has shown that for the ps–ps case at any rate such graphs are important.) This integral equation, with the above approximation and for charged scalar mesons with scalar coupling ($g_1 = g_2$; $g_3 = g_4 = 0$) is applied to the deuteron ground state, and it is found that the value of the coupling constant, $g^2/4\pi$, to fit is changed by 15% from that got using the static Yukawa potential. Owing to the uncertain importance of the neglect of other terms (fig. 8) in the kernel, this solution cannot unfortunately be considered terminal for the case of scalar mesons with scalar coupling, but it does give a good idea of the error involved (for this coupling) in the adiabatic and non-recoil approximations.

The attempts described in this section to better the adiabatic and static approximations (and thus to extend, incidentally, the region of validity of the nucleon–nucleon potential) are, momentarily, of methodological, rather than physical, interest. For the methods have not been far enough developed for a consistent attempt to be made, for instance (as in the work of Levy (1952 b)) to apply them to finding the properties of the proton–neutron system, and comparing the results with experience.

3.5

The solutions of the equations of motion in all the methods discussed so far implicitly contain the coupling constant up to all orders. For example, the static second order adiabatic potentials of § 3.2 are to be substituted into a Schrödinger equation, the exact solutions of which contain implicitly terms of second, fourth and all even order terms in the coupling constant.

Besides the methods thus far discussed the S-matrix method can also be used for the solution of nucleon–nucleon scattering problems. This has the disadvantage that in practice it is difficult to work out terms beyond fourth order in the coupling constant in the scattering S-matrix series. (As just noted, the solutions of the Schrödinger equation with a static potential contains parts corresponding to *some* terms of all orders in the S-matrix series.) On the other hand, no further approximations need be made ; and in particular the non-recoil (' static ') approximation is not made. This makes the S-matrix method more suitable for considering those high-energy collisions, where the static approximation is likely to be bad. (Watson and Lepore (1949), Beard and Bethe (1951).) However, the S-matrix method suffers in even greater degree than those of §§ 3.2, 3.3, 3.4, from the uncertainty as to the validity of the perturbation approximation.

3.6

All the methods mentioned of treating two-body interactions have been based on a perturbation expansion in the coupling constant, and a neglect, whether hypothesized or partly justified, of certain terms of higher order in the coupling constant. The strong coupling method (§ 2.1 *B*, Wentzel 1947) does not neglect such terms, but it is a static theory which completely neglects recoil and also pair creation. (This latter point probably invalidates the consideration of ps–ps coupling.)

If the strong coupling approximation method is applied to two nucleons the static self-energy of the two-body system is found to depend on the particle coordinates, and, by subtracting twice the static self-energy of one particle, the static interaction potential is deduced (Serber and Dancoff 1943, Fierz 1944, 1945). In the symmetrical s–s and ps–pv cases the potential for long separation between two nucleons in their ground states (that is, with no isobaric states excited) is just 1/9 of that given by the lowest order perturbation theory (§ 3.2). At small separations these forces become spin-independent. Serber and Dancoff (1943) conclude that it is impossible to find spin-dependent forces which, at the same time, extend to small enough separations and are of sufficient strength, to account for the properties of the deuteron.

§ 4. HEAVY NUCLEI AND NON-LINEAR MESON THEORY

In § 3 two-body forces between nucleons have been discussed and it is a possibility that these forces may be sufficient to account for the properties of heavy nuclei. Against this possibility the following points may be raised.

(*a*) The saturation property of nuclei imposes certain necessary conditions on the two-body potential. (This is illustrated, for example, in the case of charge-independent central potentials by Rosenfeld (1948 b).) It appears that a considerable repulsive interaction in odd l states in the proton–neutron potential is required. The near symmetry about 90° of the proton–neutron scattering up to 90 MEV (fig. 3), shows that up to this energy no such odd interaction exists. Thus the actual two-body forces as observed in nucleon–nucleon scattering do not seem to fulfill the saturation requirements. However, the possible modification of the two-body forces in heavy nuclei, due to the Pauli principle, must be remembered in this connection.

(*b*) Due to the strength of the meson–nucleon coupling it is not unlikely that three-body (or many-body) forces may be important. (Janossy (1939) and Primakoff and Holstein (1939) have made a preliminary study of many-body forces.)

(*c*) The success of the nuclear shell model. This is essentially an independent particle model, and it would seem that strong, short range two-nucleon forces would make the state of any one nucleon in a nucleus dependent mainly on the motions of its neighbours of the moment, and so prevent the existence of the one-body potential of an independent particle model. However, for short range two-body interactions a nucleon may not be able to follow the rapid variations in potential that are produced by the motions of its neighbours, and would then behave as if it was in a smoothed out potential. Another suggestion as to how an effective one-body potential may be produced from the two-body forces, is that the exclusion principle may suppress the effect of the short range interactions (Weisskopf 1951, Kopferman 1951).

Now an effective field theoretical treatment of heavy nuclei would require an investigation of all these points ; in particular it would require a careful study of many-body interactions. With present techniques, this seems too difficult to carry out satisfactorily. Consequently, certain authors (Schiff 1951, Malenka 1952) have been lead to discuss non-linear modifications of meson theory through which qualitative explanations of nuclear properties (in particular saturation and the independent particle model) may be obtained.

Schiff suggests that insight into the properties of a non-linear meson theory for heavy nuclei may be obtained by a classical treatment of the meson field in which the nucleons act as sources. With such a classical treatment, then, in the usual linear theory the meson field amplitude is proportional to the nucleon source strength ; and the interaction energy between a number of nucleons is equal to the sum of the pair interactions. Schiff suggests that the non-linearity be chosen so that the meson field amplitude increases less rapidly than the linearly with the nucleon source strength. Then the interaction energy between a number of nearby nucleons is less than the sum of the pair interactions. This is just the sort of effect needed to account for saturation and a stable density for heavy nuclei will be got if the attractive potential energy per nucleon increases less rapidly than the 2/3 power of the density for high densities.

The fact that in this model the attractive potential energy is less than the sum of the pair potential energies, indicates that the two-body interaction is somewhat suppressed. This smoothing out of the two-body forces leads in the direction of the independent particle picture.

Schiff takes neutral scalar mesons and considers two kinds of non-linear theory which have the following (static) meson field equations :

$$\text{(i)}\quad -\nabla^2\phi+\mu^2\phi=g\rho(\mathbf{r})\,F'(\phi), \qquad \text{(4.1)}$$

$$\text{(ii)}\quad -\nabla^2\phi+\mu^2\phi+\lambda\phi^3=g\rho(\mathbf{r}). \qquad \text{(4.2)}$$

Here $\rho(\mathbf{r})$ is the nucleon density. In (i), in accordance with the requirement that the meson amplitude increases less rapidly than linearly with the nucleon source strength $F(\phi)$ $(F'(\phi)=\partial F/\partial\phi)$ is an odd function that approaches ϕ when ϕ is small and increases less rapidly than ϕ for large ϕ.

(i) It is shown that the equation (4.1) does not lead to saturation.

(ii) By showing that g, λ can be chosen so that nuclear matter has a stable density equal to its observed value, it is shown that saturation can be achieved. The variational method gives a binding energy per nucleon of as much as 6 MeV.

Malenka (1952) considers eqn. (4.2) purely in relation to the independent particle model of the nucleus. He shows that it leads to an independent particle potential well whose shape is somewhat between that of a square well and a three-dimensional isotropic oscillator. As shown by Mayer (1950), if a strong spin orbit coupling is added, the energy levels given with such a well are sufficient to explain the magic numbers.

APPENDIX

The electromagnetic Hamiltonian density H'' is given by

$$H'' = -ie\Pi \frac{1-\tau_3}{2}(A_0-\alpha_i A_i)\Psi + \frac{e^2}{2}(\phi_1{}^2+\phi_2{}^2)\sum_{i=1}^{3} A_i{}^2$$
$$-e\sum_{i=1}^{3} A_i\left(\phi_1\frac{\partial\phi_2}{\partial x_i}-\phi_2\frac{\partial\phi_1}{\partial x_i}\right)-e(\phi_1\pi_2-\phi_2\pi_1)A_0$$
$$-ie\Pi(\alpha_i A_i)(\tau_1\phi_2-\tau_2\phi_1)f\Psi-ie\Pi(\alpha_i A_i)(\tau_1\phi_2-\tau_2\phi_1)F\Psi,$$

where $f_1=f_2=f$, $F_1=F_2=F$.

The nucleon field variables obey the commutation relations

$$[\Psi_{\varrho\sigma}(x),\Psi_{\lambda\mu}(x')]_+=0, \qquad [\Pi_{\varrho\sigma}(x)\,\Pi_{\lambda\mu}(x')]_+=0,$$
$$[\Psi_{\varrho\sigma}(x),\ \Pi_{\lambda\mu}(x')]_+=i\delta_{\lambda\mu}\delta_{\varrho\sigma}\delta(x-x').$$

The meson field variables obey the commutation relations

$$[\phi_\varrho(x),\ \phi_\sigma(x')]=0\,; \qquad [\pi_\varrho(x),\ \pi_\sigma(x')]=0,$$
$$[\phi_\varrho(x),\ \pi_\sigma(x')]=i\delta_{\varrho\sigma}\delta(x-x').$$

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Production and Scattering of Mesons

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§ 1. INTRODUCTION AND GENERAL FORMALISM

1.1. *Introduction*

SINCE the first artificial production of π-mesons at the University of California Radiation Laboratory in 1948, a large body of experimental data has been built up both regarding the production of these mesons by the interaction with nuclei of energetic protons and γ-rays, and of the subsequent interaction of the π-meson with nuclear matter. Parallel theoretical work has endeavoured to provide a unified interpretation of these experimental results. It is well known that, in no case, has this theoretical work so far proved completely satisfactory, there being yet no complete theory for any process involving mesons. Considerable success has, however, been achieved with various forms of phenomenological analysis, making use only of the meson field's transformation properties, or applying knowledge regarding the nucleon–nucleon interaction or the nucleon magnetic moments acquired from other experiments. It will be the object of this article first to review the present available evidence on the production and scattering of π-mesons : then to discuss from the analysis of this evidence whether the character of π-mesons and their coupling to nucleons can be reasonably deduced, and whether a satisfactory agreement is then found between experimental and theoretical results.

In so far as attention is confined to π-mesons the treatment can at the best only be rather incomplete. It is to be expected that account must also be given of the various heavier mesons, ζ and τ, κ and the V-particles, before the theory of π-mesons can be regarded as satisfactory. The existence of these heavy mesons, some of which can plausibly be regarded as excited virtual states of the π-meson field, or in the case of the V-particles, of the nucleon field, undoubtedly lends support to the idea of the intermediate or strong coupling of the meson–nucleon fields, giving rise to so-called isobars. It is immediately obvious, of course, that the life-times associated, for example, with the observed V-particles are so long, and the corresponding level widths so narrow, that these states can play no important role in the variation of meson production and scattering cross sections as experimentally measured. The mere existence of such states, however, lends support to the idea that comparable but much wider levels exist in terms of which an analysis of meson production experiments can be carried out in a manner analogous to that of the resonance theory of nuclear transformations. Possibly indeed this is the only form of analysis at present profitable, for example, for the phenomenon of the scattering of

π-mesons by nucleons. One is, however, justified in hoping that a more fundamental approach will eventually prove successful in treating what are after all problems of the interaction of elementary particles.

In this article attention will be principally concentrated on the elementary meson interactions, production of mesons in nucleon–nucleon collisions, by high energy quanta at nucleons and meson–nucleon scattering. The processes involving more complex nuclei on the whole add more to the knowledge of nuclear than of meson physics. There is, however, a considerable interest attaching to the problems of interaction with a few nucleons, for there the angular distribution of the products can give interesting information regarding the spins and parities of the particles concerned, and the intensities can give important information regarding the relative sign of the meson amplitude from neutrons and protons.

1.2. *Kinematics of Meson Reactions*

In the consideration of meson reactions it is frequently necessary to pass between the laboratory system, and the corresponding zero momentum system (often referred to for brevity as the c.m. system). It will prove useful here to summarize the kinematics of such transformations and tabulate some numerical values.

Suppose that in the laboratory system S a particle of mass m_1 is moving with velocity u to collide with the stationary particle m_2. Then the zero-momentum system S' has velocity v relative to S where

$$v=\gamma' u/(\alpha+\gamma'), \text{ with } \alpha=m_2/m_1,\ \gamma'=(1-u^2)^{-1/2}. \quad . \quad . \quad . \quad (1)$$

The particles to be considered are mesons, photons and nucleons. Throughout, the following notation will be used for the energy and momentum of these particles :—

photon : momentum $\mathbf{v}$, energy ν,

meson : momentum $\mathbf{k}$, energy $\omega=k_0=\sqrt{(k^2+\mu^2)}$,

proton (or neutron) momentum $\mathbf{p}$, energy $\epsilon=p_0=\sqrt{(p^2+M^2)}$.

Generally a dash will denote that the quantity is measured in the c.m. system. Natural units $\hbar=c=1$ are employed.

There are three main cases to be considered :

(i) m_1 *a Photon*, m_2 *a Proton* ($m_2=M$)

Then

$$\nu'=\left(\frac{M}{M+2\nu}\right)^{1/2}\nu,\quad v=\frac{\nu}{\nu+M},\quad \gamma=(1-v^2)^{-1/2}=\frac{(\nu+M)}{\{M(2\nu+M)\}^{1/2}}. \quad . \quad (2)$$

This applies to the case of photo-production of mesons at protons. In table 1 these quantities are given for a variety of energies of the incident photon (laboratory system) together with $T'(=\nu'+M(\gamma-1))$, the energy available for meson production in the c.m. system. Throughout we use $\mu=140$ MeV, $M=938$ MeV, and the neutron–proton mass-difference can generally be ignored.

Table 1. Photo-production of Mesons at Protons

ν (Mev)	ν' (Mev)	γ	v	T' (Mev)
160	138	1·011	0·152	148
200	167	1·016	0·175	182
250	202	1·024	0·211	223
300	234	1·031	0·242	263
350	265	1·039	0·272	302
400	294	1·048	0·298	339
500	348	1·067	0·348	410
1000	565	1·167	0·516	722

(ii) m_1 *and* m_2 *both Protons, mass* M

Then
$$\epsilon_1'=\epsilon_2'=\{M(\epsilon_1+M)/2\}^{1/2},\quad \gamma=\left(\frac{\epsilon_1+M}{2M}\right)^{1/2}. \quad . \quad . \quad . \quad . \qquad (3)$$

In table 2 some typical values are given, including again T', the energy available for meson production in proton–proton collisions. The laboratory system kinetic energy of the incident proton is denoted by T.

Table 2

T (Mev)	γ	v	T' (Mev)
300	1·0770	0·371	144
350	1·0893	0·397	167·5
400	1·1014	0·419	190
450	1·1135	0·440	213
500	1·1254	0·459	235
750	1·1831	0·534	343
1000	1·2382	0·590	447
2000	1·4374	0·718	820

(iii) m_1 *a* π*-Meson,* m_2 *a Proton*

The kinetic energy of the meson in the laboratory system is denoted T_π, and the total kinetic energy in the c.m. system, as above, by T'.

Table 3

T_π (Mev)	γ	v	T' (Mev)
50	1·006	0·114	42
100	1·014	0·166	84
150	1·022	0·207	124
200	1·031	0·242	162
300	1·049	0·303	235
500	1·089	0·396	371

The above tables illustrate the advantage at not too high energies of meson photo-production in the smaller amount of energy ‘ lost ’ in the transfer to the c.m. system. Thus, for example, if the threshold for a process in p–p collisions is 2000 Mev (laboratory system) then the corresponding γ–p threshold is about 1200 MeV. On the whole, however, it is rather meaningless to talk of the advantages of the two methods of meson production on these grounds. Experience has shown that, if a meson beam is desired in order to study the reactions produced by the meson, then the larger cross sections for nucleon–nucleon processes make a proton accelerator almost essential. Otherwise the fundamental γ–p and p–p processes are both of comparable interest, and need separate study.

The second kinematic problem that arises is the transformation of cross sections from the mass-centre to the laboratory system. For example, suppose that in the reaction $p+p\rightarrow\pi^{+}+p+n$ the differential cross section for meson production per unit energy interval per unit solid angle is known in the form

$$d\sigma=n(\omega',\theta')\,d\omega'\,d\Omega'. \quad \ldots\ldots\ldots \quad (4)$$

The transformation to the laboratory system is made by noting the invariance of $d\mathbf{k}/\omega$, so that

$$k\,d\omega\,d\Omega=k'\,d\omega'\,d\Omega'.$$

Hence

$$d\sigma=n(\omega',\theta')\left(\frac{k}{k'}\right)d\omega\,d\Omega. \quad \ldots\ldots\ldots \quad (5)$$

Here for a given ω, θ it will be necessary to find ω', k', θ' from the relations

$$\left.\begin{aligned} k'\sin\theta'&=k\sin\theta,\\ k'\cos\theta'&=\gamma(k\cos\theta-v\omega),\\ \omega'&=\gamma(\omega-vk\cos\theta). \end{aligned}\right\} \quad \ldots\ldots \quad (6)$$

When only a single energy of outgoing particle is concerned in the mass-centre system, e.g. in the reaction $\gamma+p\rightarrow\pi^{+}+n$ or $p+p\rightarrow\pi^{+}+d$ the analysis can be taken a stage further. One can now write

$$\begin{aligned} d\sigma&=f(\theta')\,\delta(\omega'-\omega_0')\,d\omega'\,d\Omega'\\ &=f(\theta')k\delta(\omega-\omega_0)\Big/\left\{k'\left(\frac{\partial\omega'}{\partial\omega}\right)_{\theta}\right\}, \end{aligned}$$

where ω_0 now depends on θ, so that

$$\frac{d\sigma}{d\Omega}=f(\theta')\frac{k^2}{\gamma k'(k-v\omega\cos\theta)}. \quad \ldots\ldots\ldots \quad (7)$$

1.3. *General Formalism and Meson Properties*

In this section some of the general properties of π-mesons will be reviewed and the interaction Hamiltonians summarized. In the first place the masses of charged and neutral π-mesons have been established as follows :

$$m(\pi^{\pm})=276\pm2m_e, \quad m(\pi^{\pm})-m(\pi^{0})=10{\cdot}5\pm2{\cdot}5m_e.$$

The mean lifetimes for the charged meson decays $\pi^{\pm} \rightarrow \mu^{\pm}+\nu$ have been established as :

$$\tau_{\pi^+}=2{\cdot}6\pm0{\cdot}1\times10^{-8}\text{ sec}, \quad \tau_{\pi^-}=2{\cdot}85\pm0{\cdot}25\times10^{-8}\text{ sec},$$

cf. for example Kraushaar (1952), Lederman *et al.* (1952). The neutral meson decay $\pi^0 \rightarrow 2\gamma$ is believed to have a life-time of the order of 10^{-14} sec.

The spin of the π^+-meson is determined from detailed balancing for the processes $p+p \rightleftharpoons \pi^+ + d$ as zero. These reactions are discussed in greater detail below where references are given. For the present it is merely noted that, within the experimental error, up to about 10% of the π^+-mesons might have spin 1. Much the simplest hypothesis is that they all have spin zero. This is consistent with the two-quantum decay observed for the π^0-meson. Neutral vector π^0-mesons, for which the two-photon decay would be absolutely forbidden, would probably decay into an electron–positron pair (the competing process is to three photons). Such pairs have not been observed, but the evidence is not strong enough to preclude the existence of any spin 1 π^0's. However, the simplest hypothesis, again, is that all π^0-mesons have spin zero.

The parity of the π-meson is determined by the π^--capture in deuterium. The experiments of Panofsky, Aamodt and Hadley (1951) show that the two processes $\pi^- + d \rightarrow 2n$ and $\pi^- + d \rightarrow 2n + \gamma$ occur with a branching ratio 2 : 1. Consideration of the time required for absorption of the meson from a p-orbit round the deuterium nucleus compared with the time for the radiative transition to an s-orbit makes it highly probable that absorption takes place from an s-orbit. In this case the occurrence of the reaction $\pi^- + d \rightarrow 2n$ means that the meson must have odd parity. The two neutrons then finish in a $^3P_1$ state. Weak coupling calculations for pseudoscalar mesons (PV coupling) indeed agree almost exactly with the experimentally observed branching ratio. These calculations are reviewed by Marshak (1951). It need only be added that more recent experience with such calculations makes this agreement seem rather fortuitous. The trend of the production and scattering evidence discussed below is to suggest that the s-wave meson interactions with nucleons are much weaker relative to the p-wave than would be predicted by weak coupling theory. However, in this case the disagreement with scalar theory is so strong that it seems most unlikely that it will be upset. The conclusion is that mesons are pseudoscalar.

For this article, therefore, the theoretical treatment will be confined to mesons of spin zero. Occasionally the predictions of scalar meson theory will be discussed for comparison, but generally it will be assumed that π-mesons are of odd parity. Both charged and neutral mesons are concerned and they can be most elegantly combined in the charge independent or symmetric theory of Kemmer (1938). In this theory the total isotopic spin of any meson and nucleon system is a constant of the motion, as discussed more fully below. The alternative so called ‘ neutral ’ coupling of neutral mesons which ascribes the same sign of coupling to the meson field of neutrons and protons will sometimes also be discussed. The interaction Hamiltonian densities are as follows.

Scalar Theory with Scalar Coupling

$$H'_{\mathrm{NM}}=g\psi^{+}\beta\phi\psi=g\bar{\psi}\phi\psi \quad \text{(neutral theory)}$$

$$=g\bar{\psi}\tau_i\phi_i\psi \quad \text{(symmetric theory)}$$

$$=g\bar{\psi}\{\tau_3\phi_3+\sqrt{2}(\tau^{-}\phi+\tau^{+}\phi^{*})\}\psi. \quad . \quad . \quad . \quad . \quad (8)$$

The isotopic spin operators are so defined that $\tau_3=+1$ acting on a neutron state, and $\tau_3=-1$ acting on a proton state. The charge exchange operators are

$$\tau^{+}=\tfrac{1}{2}(\tau_1+i\tau_2), \quad \tau^{-}=\tfrac{1}{2}(\tau_1-i\tau_2). \quad . \quad . \quad . \quad . \quad . \quad (9)$$

With the usual representation $\tau^{+}=\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}$, $\tau^{-}=\begin{pmatrix}0 & 0\\ 1 & 0\end{pmatrix}$, so that

$$\tau^{+}\psi_{\mathrm{P}}=\psi_{\mathrm{N}}, \quad \tau^{-}\psi_{\mathrm{N}}=\psi_{\mathrm{P}}.$$

The symmetrical meson field is defined by the three field functions $\phi_i (i=1, 2, 3)$, such that each ϕ is real and ϕ_1, ϕ_2 together represent the complex (charged) meson field ϕ

$$\phi=(\phi_1+i\phi_2)/\sqrt{2}, \quad \phi^{*}=(\phi_1-i\phi_2)/\sqrt{2}. \quad . \quad . \quad . \quad . \quad . \quad (10)$$

ϕ_3 represents the neutral mesons. With the conventions of sign used here ϕ has matrix elements for the absorption of positive mesons, or the emission of negative mesons. According to (8) the assumptions of symmetric meson theory are seen to be that the neutral meson coupling constant is $1/\sqrt{2}$ that of the charged mesons and opposite in sign for neutrons and protons.

There are also the interactions between the meson and nucleon and the electromagnetic field to be considered

$$\left.\begin{aligned} H'_{\mathrm{MR}}&=-ieA_0(\pi\phi-\pi^{*}\phi^{*})-ie\mathbf{A}\,.\,\{(\nabla\phi^{*})\phi-(\nabla\phi)\phi^{*}\}+e^{2}\mathbf{A}^{2}\phi^{*}\phi.\\ H'_{\mathrm{NR}}&=e\bar{\psi}\,\frac{(1-\tau_3)}{2}\,(-i\boldsymbol{\gamma}\,.\,\mathbf{A}+\gamma_4A_0)\psi.\end{aligned}\right\} \quad . \quad . \quad (11)$$

Here π is the momentum conjugate to ϕ. In symmetric theory

$$\pi=(\pi_1-i\pi_2)/\sqrt{2}.$$

Pseudoscalar Theory (pseudoscalar coupling)

Only the symmetric theory expression is given :

$$H'_{\mathrm{NM}}=ig\bar{\psi}\gamma_5\tau_i\phi_i\psi=g\psi^{+}\rho_2\tau_i\phi_i\psi. \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

Pseudoscalar Theory (pseudovector coupling)

$$H'_{\mathrm{NM}}=\frac{if}{\mu}\,\bar{\psi}\gamma_5\gamma_\nu\partial_\nu(\tau_i\phi_i)\psi$$

$$=\frac{f}{\mu}\,\psi^{+}\{\rho_1\tau_i\pi_i+\boldsymbol{\sigma}\,.\,\nabla\tau_i\phi_i\}\psi. \quad . \quad . \quad . \quad . \quad (13)$$

$$H'_{\mathrm{NMR}}=\frac{ief}{\mu}\,\bar{\psi}\gamma_5\gamma_kA_k(\tau_1\phi_2-\tau_2\phi_1)\psi. \quad . \quad . \quad . \quad . \quad (14)$$

The pseudoscalar mesons have also the same direct coupling to the radiation field as scalar mesons.

Throughout this article, unless where otherwise stated, the meson coupling constants g (or f) will be in the rationalized symmetric theory units defined above. The ' ordinary ' charged theory constants, G say, are related to these by $G^2=g^2/(2\pi)$. The Hamiltonian density for the free meson field is

$$H_{\mathrm{M}}=\tfrac{1}{2}\{\pi_i^{\,2}+(\nabla\phi_i)^2+\mu^2\phi_i^{\,2}\}. \qquad (15)$$

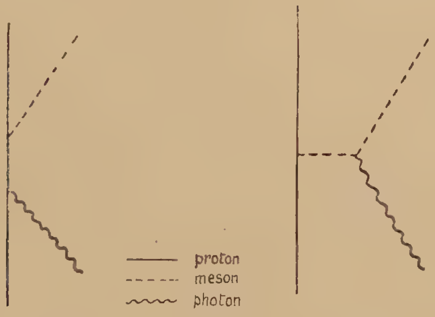
A partial equivalence may be shown between the PS and PV coupling for pseudoscalar mesons, cf. Dyson (1948) and Case (1949). If the Hamiltonian for pseudoscalar coupling is subjected to the transformation $H'=\mathrm{e}^{iS}H\mathrm{e}^{-iS}$, with

$$S=\frac{g}{2M}\int\bar{\psi}\gamma_4\gamma_5\tau_i\phi_i\psi\,d\mathbf{r},$$

then the transformed interaction terms are

$$\frac{g}{2M}\int\psi^+[(\boldsymbol{\sigma}\cdot\nabla)\tau_i\phi_i+\rho_1\tau_i\pi_i-e\boldsymbol{\sigma}\cdot\mathbf{A}(\tau_1\phi_2-\tau_2\phi_1)]\psi\,d\mathbf{r}$$
$$+M\int\psi^+\gamma_4\left[\left(1+\frac{ig}{M}\gamma_5\tau_i\phi_i\right)\exp\left(-\frac{ig}{M}\gamma_5\tau_i\phi_i\right)-1\right]\psi\,d\mathbf{r}$$
$$+\text{terms of order } g^2/M^2. \qquad (16)$$

Fig. 1



(*a*) (*b*)

Feynman diagrams for photo-meson production.

§ 2. PRODUCTION OF MESONS BY PROTONS

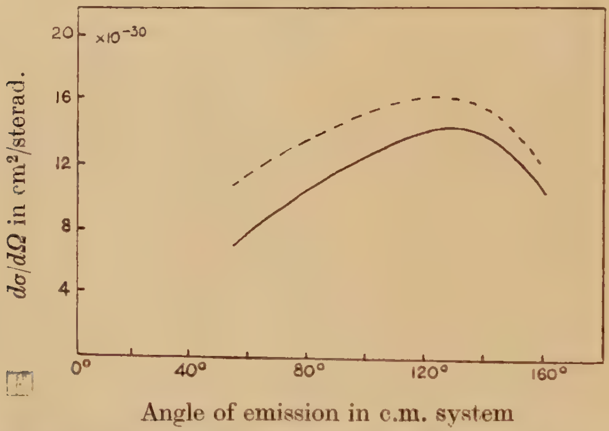
It is to be expected that the production of mesons by photons, at least in the fundamental cases of production at single nucleons, will prove one of the most informative processes regarding the character of mesons and their coupling to nucleons. Typical lowest order Feynman diagrams for the process are sketched in fig. 1 (clearly only fig. 1 (*a*) applies in the case of neutral meson production). In these two-vertex diagrams the process occurring at the one vertex involving the electromagnetic interaction is certainly one adequately treated by perturbation theory, and uncertainty should then be reduced to the treatment of the meson–nucleon interaction. The expectation is at least to some extent justified but difficulties remain both in how to give an account of the reasonably strong meson–nucleon interaction and in how to include magnetic moment interactions in case (*a*). This latter becomes particularly important in the consideration of neutral meson production.

2.1. *Production in Hydrogen*

(*a*) *Positive Mesons*

One of the inevitable difficulties associated with the study of photoproduction of mesons is the bremsstrahlung spectrum of photons present in the primary beam. In the reaction $\gamma+p\rightarrow\pi^{+}+n$ it is possible, however, to identify the energy of the photon producing a given meson if, for example, the meson energy and angle are measured in the laboratory system. The most extensive experiments on this reaction have been carried out at Berkeley by Steinberger and Bishop (1952). They used scintillation counters to detect the π^{+}-mesons, discriminating against other particles by requiring a delayed coincidence with the decay positron emitted after the π^{+} is brought to rest. The meson energy at any angle was selected by varying the absorber thickness in front of the crystal. The results so found have then to be corrected for loss of mesons through absorption in flight.

Fig. 2



Angular distribution of π^{+}-mesons from 250 Mev photons on hydrogen. (Dotted curve includes correction for a meson absorption cross section equal to the nuclear area.)

Particular attention was paid to two aspects of the process. Firstly the angular distribution of mesons produced by photons of 255 Mev in the laboratory system (204 Mev in the c.m. system) was determined. This is shown in the c.m. system in fig. 2.* The total cross section is about $1{\cdot}6\pm0{\cdot}3\times10^{-28}$ cm^{2}.

Secondly the excitation curve for the production of mesons at 90° in the laboratory system was measured against the photon energy. This curve is shown in fig. 3. It appears to show an almost linear rise with the excess photon energy near the threshold, but to vary more like the square root of this quantity at higher energies.

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* The numerical values in fig. 2 are as in Madey, U.C.R.L. 1856.

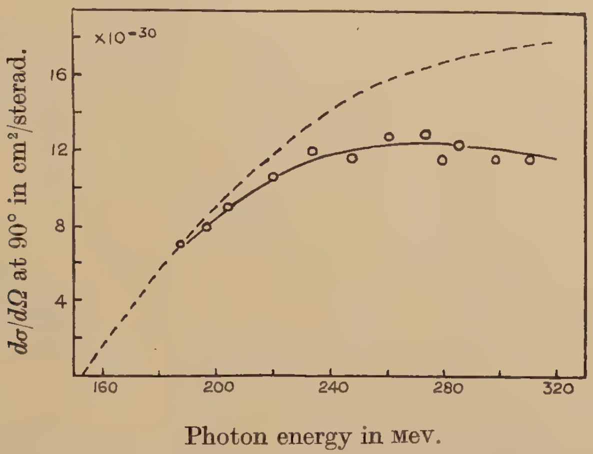
(*b*) *Neutral Mesons*

The detection of the neutral mesons produced in the reaction

$$\gamma + p \rightarrow \pi^0 + p$$

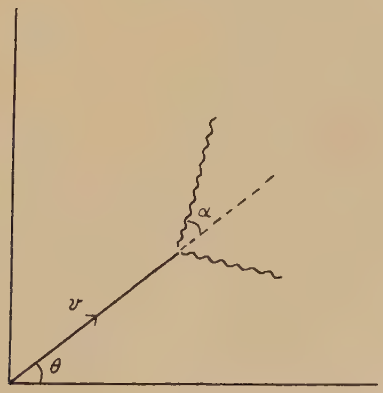
has to be made through the decay quanta of the π^0-meson. The experiment is, therefore, less direct than that for π^+-production. It is perhaps useful first to study briefly the kinematics of the reactions involved.

Fig. 3



Excitation curve for π^+-production by photons on hydrogen. (Dotted curve includes absorption correction as in fig. 2.)

Fig. 4



Kinematics of neutral meson decay.

Suppose that in the mass-centre system neutral mesons of a single velocity v are being produced with some unknown angular distribution.

The decay quanta are isotropic in the π^0-rest system. Hence it may be deduced that the probability from a given meson for a decay photon to go into solid angle $d\Omega_\alpha$ at angle α as shown in fig. 4 is

$$p(\alpha)\,d\Omega_\alpha=\frac{d\Omega_\alpha}{2\pi\gamma^2(1-v\cos\alpha)^2},\qquad \gamma=(1-v^2)^{-1/2}. \quad . \quad . \quad (17)$$

If, then, the neutral meson production cross section has in S' the form

$$\frac{d\sigma(\pi^0)}{d\Omega}=f(\theta), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

the differential cross section for the decay quanta will be

$$\frac{d\sigma(\gamma)}{d\Omega}=\int\frac{d\sigma(\pi^0)}{d\Omega'}\,p(\alpha)\,d\Omega',$$

where α is the angle between the directions specified by the elements of solid angle $d\Omega$, $d\Omega'$. The integral is easily carried out if $d\sigma(\pi^0)/d\Omega$, and $p(\alpha)$ are expressed in terms of Legendre polynomials. If

$$\frac{d\sigma(\pi^0)}{d\Omega}=\Sigma A_lP_l\,(\cos\theta)\,;\;\; p(\alpha)=\Sigma B_lP_l\,(\cos\alpha), \quad . \quad . \quad . \quad (19)$$

then

$$\frac{d\sigma(\gamma)}{d\Omega}=4\pi\sum_l\frac{A_lB_l}{(2l+1)}P_l\,(\cos\theta). \quad . \quad . \quad . \quad . \quad . \quad (20)$$

The coefficients B_l for the Legendre expansion of $p(\alpha)$ are all quite directly expressible in terms of elementary functions.

For the case of photo-production of π^0-mesons, unfortunately, there will be a spectrum of velocities v, so that direct conclusions cannot be drawn from the angular distribution of the decay photons. The above method can, however, be applied to the analysis of π^--charge exchange scattering in hydrogen (cf. § 4.1 below).

A second quantity of interest for the π^0-decay photons is the 'correlation angle' ϕ, say, between the directions of emission of any pair of photons. For the case shown in fig. 4,

$$\sin\phi/2=(\cos^2\alpha+\gamma^2\sin^2\alpha)^{-1/2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

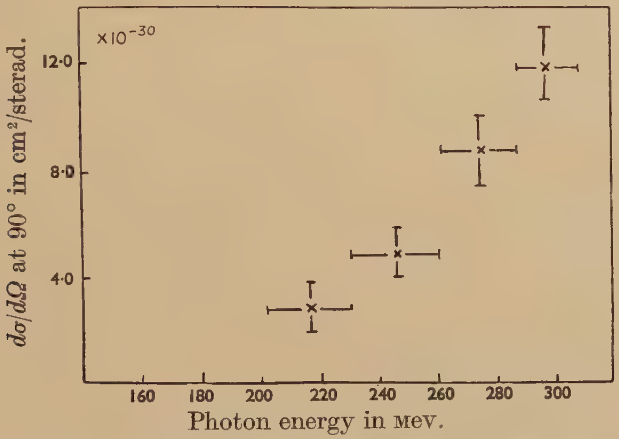
from which using (17) one finds for the probability $P(\phi)\,d\phi$ of finding a correlation angle in the interval ϕ, $\phi+d\phi$

$$P(\phi)\,d\phi=\frac{\sin\phi}{(1-\cos\phi)^{3/2}}\,\frac{d\phi}{v[(1-\cos\phi)\gamma^2-2]^{1/2}}. \quad . \quad . \quad (22)$$

This correlation probability has the property of being strongly peaked towards the minimum angle $\phi_{\min}(=2\cos^{-1}v)$ particularly for high meson energies.

Measurements of neutral meson photo-production in beryllium and hydrogen have been made by Panofsky *et al.* (1952), using this correlation angle technique. In hydrogen only an approximate cross section could be obtained, though it could be shown that the excitation curve increased with a higher power of the photon excess energy above threshold than in the corresponding π^+-production. More direct measurements have been made at Cornell by Silverman and Stearns (1951, 1952). In their experiments the recoil proton at a given angle, generally 30°, was detected, and had its energy measured in a thick NaI crystal. If the incident photon has an energy in the region 200–300 Mev, then corresponding to the 30° proton the neutral meson will be produced at $90° \pm 10°$. A high energy γ-ray detector was, therefore, placed to detect the decay quanta produced in this direction, which will be the one of maximum γ-ray intensity according to (17). Any coincidence between these two counters then denotes a π^0-production event, the energy of the responsible incident photon being determined by the measurement of the proton recoil energy. This latter varies from about 30 Mev for incident photon energy $\nu = 200$ Mev, to 60 Mev at $\nu = 300$ Mev. The results obtained for excitation at 90° are shown in fig. 5. The data can be roughly fitted by a power law $d\sigma/d\Omega \propto (\nu - \mu)^{1\cdot 8 \pm 0\cdot 5}$. The corresponding exponent for π^+-production lies between $\frac{1}{2}$ and 1.

Fig. 5



Excitation curve for neutral meson production by photons on hydrogen.

Recent measurements by Cocconi and Silverman* give information about the angular distribution of π^0-production. The differential cross section falls off steeply between meson angles 45° and 135° for incident photons with mean energy about 290 Mev. The variation is consistent with $d\sigma/d\Omega \propto (2 + 3 \sin^2 \theta)$ in the c.m. system.

2.2. *Photo-Meson Production at Deuterium*

The analysis of experiments on photo-production at deuterium presents more difficulties, for now, with a given incident photon energy and at a given angle, there is in general a broad spectrum of mesons produced. Relatively straightforward of interpretation, however, is the ratio of the production cross sections for negative and positive mesons. The two processes concerned are $\gamma + \mathrm{d} \rightarrow \pi^+ + 2\mathrm{n}$ and $\gamma + \mathrm{d} \rightarrow \pi^- + 2\mathrm{p}$. If charge independence of nuclear forces is assumed, so that, ignoring Coulomb effects, the same wave-function describes the 2n and the 2p system, then the π^-/π^+ ratio will approximately determine the ratio of the cross sections for the processes $\gamma+n\rightarrow\pi^-+p$, and $\gamma+p\rightarrow\pi^++n$. The experimental result is a ratio equal to unity within the experimental error. The most extensive measurements have been made by White *et al.* (1952). They used a deuterium gas target and photographic emulsion detection. Their results are presented in table 4.

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* Reported at Copenhagen Conference, June 1952, by R. R. Wilson.

Table 4. Ratio of π^-/π^+ Mesons from Deuterium

Angle of production	T_π (MEV)	π^-/π^+ Ratio
45°	43	0·79±0·21
	57	1·04±0·16
	87	0·89±0·21
	128	0·99±0·30
90°	34	1·24±0·20
	70	0·98±0·14
135°	39	1·37±0·22
	57	1·05±0·18

The mean over all angles is 1·07±0·07. Results consistent with the above have also been found by Littauer and Walker (1951), and Lebow *et al.* (1952).

The ratio of positive meson production in deuterium compared with that in hydrogen has also been measured by White *et al.* (1952). In the case of production in deuterium the final di-neutron system is restricted to the ^1S, ^3P . . . states. At low energies, or, to be more precise, when only a low relative kinetic energy is available for the two neutrons, the ^1S state would be favoured. It can, however, only be reached if the proton spin is changed in the meson production. Measurements of the $\pi^+(D)/\pi^+(H)$ ratio may, therefore, be expected to give information on the spin dependence of the meson producing interaction. At various angles the differential cross section for production of mesons in given energy intervals by the Berkeley electron synchrotron 318±10 MEV beam were measured with results shown in table 5. Similar data are given by Lebow *et al.* (1952), but with rather greater statistical errors. These results are discussed more fully in § 3.2 below.

Table 5

Angle of production	Meson energy interval (MEV)	Incident photons responsible (MEV)	$\pi^+(D)/\pi^+(H)$
45	38–134	190–300	0·74±0·08
90	39–93	210–300	0·74±0·05
135	37–67	230–300	0·79±0·06

No detailed measurements are as yet available for the production in deuterium of neutral mesons but preliminary values, measured by Cocconi and Silverman,* give, at 90° in the laboratory system, a ratio of approximately 2 for such mesons, summed over all energies, compared with the corresponding production in hydrogen, for the Cornell synchrotron photon beam. At 135° the ratio is nearer 1·5.

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* These measurements have now been published, Cocconi and Silverman (1952).

2.3. *Photo-production at Complex Nuclei*

On the whole the photo-production of mesons at complex nuclei tends to be a process more informative on nuclear properties than on fundamental meson interactions. There are, however, two general fields of investigation which call for description—the variation with atomic number of the ratio of the yields of negative and positive mesons, and the same variation for the total number of charged or neutral mesons.

Table 6. Yield of Charged Photo-mesons

Element	A	Z	Yield of $\pi^\pm$	π^-/π^+
H	1	1	1·04±0·07	
D	2	1	1·91±0·10	1·19±0·12
Li	7	3	6·90±0·31	2·06±0·18
Be	9	4	7·78±0·03	2·25±0·11
C	12	6	7·31±0·08	1·06±0·02
O	16	8	8·33±0·18	1·04±0·05
F	19	9	10·3 ±0·18	1·42±0·05
A	27	13	13·2 ±0·29	4·20±0·05
P	31	15	14·8 ±0·4	1·04±0·06
S	32	16	14·4 ±0·24	0·85±0·03
K	39	19	19·0 ±0·6	1·02±0·1
Ca	40	20	14·2 ±0·7	0·58±0·06
Mn	55	25	20·0 ±0·9	1·20±0·11
Co	59	27	22·9 ±0·5	1·34±0·07
As	75	33	23·6 ±0·9	1·43±0·1
I	127	53	39·0 ±1·1	1·46±0·08
Bi	209	83	44·5 ±2·0	1·32±0·12

The π^-/π^+ ratio has been the subject of several investigations. The early experiments of McMillan *et al.* (1950) and Peterson *et al.* (1950) established a value $\simeq 1{\cdot}3$ for carbon fairly independently of energy and angle of observation. Littauer and Walker (1952), using the Cornell synchrotron, made extensive measurements of the yield at 135° of mesons in the energy range 65±15 MeV. For symmetrical ($N=Z$) nuclei they found the π^-/π^+ ratio decreasing steadily with atomic number from 1·19±0·12 for ^{2}H to 0·58±0·06 for ^{40}Ca. Nuclei with excess neutrons show larger ratios, particularly high being 2.2 for ^{9}Be and 1·9 for ^{7}Li. Littauer and Walker found a striking correlation between the π^-/π^+ ratio and the energy difference $M_{Z-1}-M_{Z+1}$ of the neighbouring isobars of charge $(Z-1)$ and $(Z+1)$. This is the energy difference between the final states of the nucleus reached in the two cases. Clearly, therefore, it does give some measure of the relative probability with which a neutron can be changed to a proton leading to π^--production, or, alternatively, a proton can be changed to a neutron. Quantitative treatment is difficult but the general conclusion is that a loosely bound particle participates strongly in charged meson production, particularly if energy considerations favour its change of charge. The dominance of these factors is shown by the considerable increase (to about 3·7) of the π^-/π^+ ratio for ^{9}Be obtained by Littauer and Walker when the incident photon beam maximum energy was lowered to 256 MeV. Results obtained by Medicus (1951) and Carothers (1952) have given more detailed information on the ^{9}Be ratio, and its angular dependence.

Regarding the total yield of charged and of neutral mesons the experiments of Littauer and Walker (1952) and Steinberger *et al.* (1950), and Panofsky *et al.* (1952) respectively, have demonstrated an approximate $A^{2/3}$ dependence. The results for charged mesons are shown in table 6.

§ 3. THEORY OF PHOTO-MESON PRODUCTION

3.1. *Production in Hydrogen*

Weak coupling calculations have been carried out by many authors for the production of mesons in hydrogen. The process $\gamma + \mathrm{p} \rightarrow \mathrm{n} + \pi^+$ has been considered in particular by Araki (1950) and Brueckner (1950). With the interaction Hamiltonians of § 1.3 the lowest order calculations are carried out quite directly using the Feynman technique. Ignoring various relativistic corrections one finds, near the threshold, for scalar and pseudo-scalar mesons (in the laboratory system)

$$\frac{d\sigma_{\mathrm{s}}}{d\Omega} = \frac{e^2 g^2}{4\pi} \frac{k^3 \sin^3\theta}{\omega^2 \nu^3 (1 - v\cos\theta)^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (23)$$

$$\frac{d\sigma_{\mathrm{ps}}}{d\Omega} = \frac{e^2 f^2}{2\pi} \frac{1}{\mu^2} \frac{k}{\nu} \left[1 - \frac{\mu^2 k^2 \sin^2\theta}{2\omega^2 \nu^2 (1 - v\cos\theta)^2} \right]. \quad . \quad . \quad . \quad . \qquad (24)$$

Here, as elsewhere, the meson momentum and energy are denoted by k, ω, and the meson velocity by v. ν denotes the incident quantum energy. To order $e^2 g^2$ the PS and PV couplings for pseudoscalar theory give identical results.

One characteristic feature of (23) and (24) is the dipole-like angular distribution for the scalar mesons with the maximum thrown forward by the $(1 - v\cos\theta)^{-2}$ factor, contrasted with the more isotropic pseudoscalar distribution. The other main difference between the two theories is in the shape of the excitation curve—the k^3 factor in the scalar meson case corresponding with the production of p-wave mesons, whilst s-waves are permitted in pseudoscalar theory. It could not be said that either theory is in particularly close agreement with the facts, particularly violent disagreement being shown with the angular distribution predicted by scalar theory.

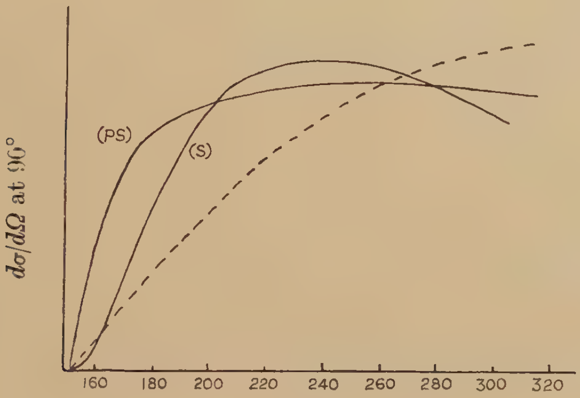
A certain degree of qualitative support to the angular distributions predicted by weak coupling theory is provided by the intermediate and strong coupling calculations of Watson and Hart (1950) and Fujimoto and Miyazawa (1950). So long as the possible occurrence of isobars is neglected the weak coupling results are reproduced exactly in shape, though with a reduction factor depending on the strength of the coupling, and becoming, for example, in the pseudoscalar case, 1/6 in the limit of very strong coupling. This result is less surprising since the same considerations of angular momentum and parity apply in both cases.

Some interesting considerations on the ratio of the processes $\gamma+\mathrm{n}\rightarrow\pi^-+\mathrm{p}$ and $\gamma+\mathrm{p}\rightarrow\pi^++\mathrm{n}$ are provided by Brueckner and Goldberger (1949). For the first process the incident photon has two currents with which to interact, that carried by the π^--meson and that carried by the recoil proton. In the second process there is only the meson current. If these currents are those associated with the movement of the particles, then allowing for retardation one might expect

$$\frac{\sigma(\pi^+)}{\sigma(\pi^-)}=\left(\frac{e\mathbf{v}^+\cdot\mathbf{A}}{1-v^+\cos\theta}\right)^2\Big/\left\{-\frac{e\mathbf{v}^-\cdot\mathbf{A}}{1-v^-\cos\theta}+\frac{e\mathbf{v}_\mathrm{p}\cdot\mathbf{A}}{1-v_\mathrm{p}\cos\phi}\right\}^2 \quad . \quad (25)$$

where $\mathbf{v}^\pm$, θ are the meson velocity and angle of production, $\mathbf{v}_\mathrm{p}$, ϕ are the corresponding quantities for the recoil proton. This would lead to a much too large π^-/π^+ ratio to accord with the deuterium production experimental evidence, e.g. $\sigma(\pi^-)/\sigma(\pi^+)=1{\cdot}55$ for $T_\pi=40$ MeV at 90° in the laboratory system. The alternative assumption of a dominant photon interaction with magnetic moments makes the ratio $\sigma(\pi^-)/\sigma(\pi^+)$ very nearly equal to unity.

Fig. 6



Photon energy in MeV.

Theoretical excitation curve for π^+-production by photons on hydrogen.

The production of neutral mesons provides further theoretical difficulties. As has been noted, the reaction $\gamma+\mathrm{p}\rightarrow\pi^0+\mathrm{p}$ becomes comparable with the π^+-production at an incident photon energy of about 200 MeV. Weak coupling calculations, on account of the suppression of graph (*b*), fig. 1 in this case, would predict a cross section reduced by a factor $(\mu/M)^2$. To overcome the difficulty it was proposed by Kaplon (1951) and Aizu *et al.* (1951) to include a Pauli term of the form $\sigma_{\mu\nu}F_{\mu\nu}$ in the nucleon–electromagnetic interaction, thus taking account of the static nucleon-magnetic moments. A considerable improvement is achieved in this way, but with the neutral and charged coupling constants related, say, as in symmetrical meson theory the neutral meson cross section remains too small in the region $\nu=200$–300 MeV. Furthermore the angular distribution of the neutral mesons is rather isotropic in the laboratory system in contradiction with experiment.

More recently calculations have been made by Koba *et al.* (1951) in the case of charged mesons, and by Minami (1951) in the case of neutral mesons, to assess the importance of higher order corrections to the perturbation calculations. The calculations for charged mesons are carried out to order e^2g^4 and for neutral to order e^2g^6, in both cases for pseudoscalar mesons. Very great complications are involved, only a little alleviated by the use of gauge invariance to classify the permissible types of matrix element, and no very certain conclusions can be drawn. Indeed, it is idle to hope for a satisfactory perturbation theory treatment of photo-meson production when one does not exist for the anomalous nucleon magnetic moments. The fourth order calculations, however, do suggest the invalidity of the use of the static anomalous moments in Pauli terms, revealing as they do considerable non-static anomalous moment corrections.

An alternative approach to photo-meson production at nucleons is to make explicit appeal to the existence of isobars in a strong coupling theory. This has been done for the case of pseudoscalar mesons by Fujimoto and Miyazawa (1950) and by Brueckner and Case (1951). The former authors make a quantum mechanical strong coupling calculation. Photo-meson production can now proceed through the excitation of the nucleon by capture of the incident photon to a $J=3/2$, $T=3/2$ isobaric state (cf. § 4.1 below), which then decays with production of either a neutral or charged meson. If this is the only intermediate state involved then the angular distribution of the emitted mesons has the form $(2+3\sin^2\theta)$ in the c.m. system, in agreement with the rather scanty experimental information. In detail one finds

$$\left.\begin{aligned}\frac{d\sigma(\pi^0)}{d\Omega}&=\frac{1}{144}\,\frac{e^2f^2}{4\pi\mu^2}\cdot\frac{k^3}{4\nu}\,\frac{2+3\sin^2\theta}{(\nu-\nu_0)^2+\frac{1}{4}\Gamma^2},\\ \frac{d\sigma(\pi^+)}{d\Omega}&=\frac{1}{2}\,\frac{d\sigma(\pi^0)}{d\Omega},\end{aligned}\right\}\quad . \quad . \quad . \quad . \quad . \qquad (26)$$

where ν, ν_0 denote respectively the incident energy, and the excitation energy of the isobar, and Γ is the isobar level width. Brueckner and Case (1951) give a classical strong coupling treatment, i.e. treating the nucleon spin and the isotopic spin as classical spin vectors. This is known in the comparable problem of nucleon–meson scattering to yield results not far different from the quantum mechanical strong coupling method (cf. Pauli (1946)).

For charged mesons a second contribution to the photo-production comes from the interaction of the incident photon with the meson current

$$\frac{d\sigma}{d\Omega}=\frac{2}{9}\,\frac{e^2f^2}{2\pi}\,\frac{1}{\mu^2}\,\frac{k}{\nu}\left[1-\frac{2k^2\sin^2\theta}{\{(\mathbf{k}-\boldsymbol{\nu})^2+\mu^2\}^2}\right]. \quad . \quad . \quad . \quad . \quad . \qquad (27)$$

This is identical to the form found in perturbation theory with the neglect of nucleon recoil, apart from the factor 2/9 resulting from the classical averaging over the spin variables.

It is a short step from these strong coupling treatments making use of resonant isobaric states to a thorough going phenomenological analysis rather along the lines of nuclear transformation theory. Such a treatment has been given by Brueckner and Watson (1952). The s-wave meson interaction is now included as well as the p-wave. The meson–nucleon system can thus exist in $J=1/2$ or $J=3/2$ states with isotopic spin $T=1/2$ or $T=3/2$ (cf. § 4.1). The assumption, based on scattering evidence, is made that interaction is strong in the $J=3/2$, $T=3/2$ state giving rise to a resonance at energy E_{R}. Also, to accord with the shape of the π^0-production excitation curve, it is assumed that the total s-state contribution for neutral mesons vanishes. This involves a balance between the amplitudes for this contribution arising from the $T=1/2$, $T=3/2$ states. If the transition elements for interaction with the incident photons are taken from weak coupling theory, the cross sections are then completely determined, so far as they come only from the $J=1/2$, $T=1/2$ or 3/2 and $J=3/2$, $T=3/2$ states.

$$\left.\begin{aligned}\sigma(\pi^+)&=e^2f^2\frac{k}{\nu}\frac{1}{\mu^2}+\tfrac{1}{2}\sigma(\pi^0),\\ \sigma(\pi^0)&=e^2f^2\frac{k}{\nu}\frac{1}{\mu^2}\frac{4}{81}\frac{k^2\nu^2}{M^2\mu^2}\frac{(\gamma_{\mathrm{n}}-\gamma_{\mathrm{p}})^2}{\left(1-\dfrac{E}{E_{\mathrm{R}}}\right)^2+\dfrac{\Gamma^2}{4E_{\mathrm{R}}{}^2}},\end{aligned}\right\} \quad . \quad . \quad (28)$$

where E_{R}, Γ_{R} denote the resonance level and its width, and γ_{n}, γ_{p} denote the nucleon magnetic moments in nuclear magnetons. The rest of the notation is as used previously. Good agreement with the experimental excitation curves can be secured with a formula of this type, particularly if ' non-static ' values of the nucleon magnetic moments are introduced, say about one and a half times the static ones.

One doubtful element in the above analysis is the relation demanded between the s-wave meson interactions in the two $J=1/2$ states. The scattering evidence discussed in § 4.1 makes it seem likely that the $T=1/2$, $S_{1/2}$ interaction is much weaker than that for $T=1/2$, $S_{3/2}$. Also the use of the one level Breit–Wigner formula for the energy variation of the transition elements seems to take this formalism beyond its range of validity. The fact remains, however, that a partial wave analysis of the low energy photo-meson production data is clearly valid. Such an analysis is not completely arbitrary since it has to be consistent with evidence on meson–nucleon scattering. Further it provides a considerable help in the physical appreciation of the character of meson interactions. The danger of the method seems to rest in the temptation to form too explicit a model. In particular there is as yet no real justification to talk of resonance levels. The rise with energy of the cross section curves for photo-meson production is dictated by the meson partial waves involved.

The flattening off and possible fall of the cross sections can be ascribed to the effect of radiation damping without any mention of isobars.

3.2. *Theory of Photo-production at Complex Nuclei*

Several theoretical treatments have been proposed for the problem of photo-meson production at complex nuclei. These treatments have all made use of the ‘ impulse approximation ’—that is they have assumed that the meson production amplitudes for the single nucleons, treated as free but with the correct momentum distribution, are linearly superposed to form that for the complete nucleus. The validity of the approximation is hard to assess on account of one's lack of knowledge of the state of the meson field within the nucleus and how this may be affected by non-linearity. However, the other requirements of the impulse approximation—‘ collision time ’ short compared with the periods of the nuclear system, individual amplitudes small compared with the distance between sources—are well enough met. The matrix element for, say, positive meson production is assumed expressed in the form

$$\langle \mathrm{n} \,|\, T(\mathbf{r},\, \mathbf{p},\, \boldsymbol{\sigma},\, \boldsymbol{\tau}) \,|\, \mathrm{p} \rangle$$

where $|\mathrm{n}\rangle$, $|\mathrm{p}\rangle$ denote the final neutron and initial proton states. Allowing for momentum conservation, and reducing any Dirac operators occurring to non-relativistic ones, T may be expressed in the form

$$T = \exp\,[i(\boldsymbol{\nu}-\mathbf{k}) \,.\, \mathbf{r}]\, \tau^{+}(\boldsymbol{\sigma} \,.\, \mathbf{K}+L). \quad . \quad . \quad . \quad . \quad . \quad (29)$$

Here $\boldsymbol{\nu}$, $\mathbf{k}$ are the incident photon and meson momenta, τ^{+} converts proton to neutron and $\mathbf{K}$, L depend on the momenta of the particles concerned or the photon polarization. Perturbation theory suggests in fact that $\mathbf{K}$, L are independent of the nuclear momenta.

For production at a free proton the cross-section is given by

$$d\sigma_{\mathrm{p}} = \frac{(2\pi)^{-2}}{(1+p/p_0)}\, d\mathbf{k} \int d\mathbf{n}\; (K^2+L^2)\delta(\mathbf{n}+\mathbf{k}-\boldsymbol{\nu}-\mathbf{p})\delta(n_0+k_0-\nu_0-p_0) \quad . \quad (30)$$

where $\mathbf{n}$, $\mathbf{p}$ denote the neutron and proton momenta, and n_0, p_0, k_0, ν_0 denote the corresponding particle energies. For this section the continuum wave functions are normalized to a density $1/(2\pi)^3$ at infinity. For production at a stationary proton the differential cross section may be written

$$\frac{d\sigma_{\mathrm{p}}}{d\Omega} = (2\pi)^{-2} k k_0 (K^2+L^2) \frac{k n_0}{[k n_0 + k_0(k-\nu\cos\theta)]} \quad . \quad . \quad . \quad (31)$$

where k_0, n_0 are determined in terms of ν, θ by momentum and energy conservation.

The case of π^{+}-production in deuterium will first be considered, for here the impulse approximation may be expected to be most valid. Treatments have been given by Machida and Tamura (1951), Morpurgo (1950), Chew and Lewis (1951) and Lax and Feshbach (1952). The initial deuteron will be assumed at rest. The differential cross section may then be written

$$d\sigma = (2\pi)^{-2}\, d\mathbf{k} \iint |Q|^2\, d\mathbf{D}\, d\mathbf{P}\; \delta(W_{\mathrm{D}}+k_0+P^2/M+D^2/(4M)-\nu_0) \quad . \quad (32)$$

where Q is the matrix element for the transition, and, apart from notation previously used in this section, $\mathbf{D}$ is the centre of mass momentum of the final 2-neutron system and $2\mathbf{P}$ is their final relative momentum, W_D is the deuteron binding energy. In accordance with the impulse approximation

$$Q=\langle f|T_1+T_2|i\rangle,$$

where

$$T_i=(\boldsymbol{\sigma}^{(i)}.\mathbf{K}+L)\exp(i\mathbf{q}.\mathbf{r}_i)\tau^{(i)+}. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (33)$$

$\mathbf{q}=\boldsymbol{\nu}-\mathbf{k}$ denotes the momentum transferred to the nucleon system. The calculation of the matrix elements proceeds exactly as in the case of nucleon–nucleon meson production discussed in § 5.2 below. For the initial and final states of the nuclear system one has

$$\left.\begin{aligned} |i\rangle &={}^1(\tau)_0\,{}^3(\sigma)_m(1/2\pi)^{3/2}v(r),\\ |f\rangle_e&={}^3(\tau)_1\,{}^1(\sigma)_0u_e(\mathbf{r})(2\pi)^{-3/2}\exp(i\mathbf{D}.\mathbf{R}),\\ |f\rangle_0&={}^3(\tau)_1\,{}^3(\sigma)_{m'}u_0(\mathbf{r})(2\pi)^{-3/2}\exp(i\mathbf{D}.\mathbf{R}).\end{aligned}\right\} \quad . \quad . \quad . \qquad (34)$$

Here $\mathbf{R}=\frac{1}{2}(\mathbf{r}_1+\mathbf{r}_2)$, $\mathbf{r}=(\mathbf{r}_1-\mathbf{r}_2)$, $v(r)$ denotes the deuteron ground state wave function. The final state may either be a spin singlet with even parity denoted $|f\rangle_e$ or a spin triplet with odd parity denoted $|f\rangle_0$. The respective matrix elements are

$$Q_e=1/\sqrt{2}\langle{}^1(\sigma)_0|(\boldsymbol{\sigma}^{(1)}-\boldsymbol{\sigma}^{(2)}).\mathbf{K}|{}^3(\sigma)_m\rangle E, \quad . \quad . \quad . \quad . \quad . \qquad (35)$$

$$Q_0=i/\sqrt{2}\langle{}^3(\sigma)_{m'}|(\boldsymbol{\sigma}^{(1)}+\boldsymbol{\sigma}^{(2)}).\mathbf{K}+2L|{}^3(\sigma)_m\rangle O, \quad . \quad . \quad . \quad . \qquad (36)$$

with $E=\int u_e{}^*\cos(\frac{1}{2}\mathbf{q}.\mathbf{r})v\,d\mathbf{r}$, $\qquad O=\int u_0{}^*\sin(\frac{1}{2}\mathbf{q}.\mathbf{r})v\,d\mathbf{r}$.

The first matrix element corresponds with transitions in which there is a nuclear spin flip. Such transitions would not be allowed if $\mathbf{K}=0$, then there would be no s-state available for the final two neutron systems, the ^{3}P state being the lowest partial wave available. In this case, near the threshold, or when nearly all the available energy is taken by the meson one would expect the π^+-production from deuterium to be reduced relative to that in hydrogen.

When the spin summations and averages are carried out the cross section appears in the final form

$$d\sigma=(2\pi)^{-2}d\mathbf{k}\int d\mathbf{P}[\tfrac{2}{3}E^2|\mathbf{K}|^2+O^2\{\tfrac{4}{3}|\mathbf{K}|^2+2|L|^2\}]\delta(E_f-E_i) \quad . \quad . \qquad (37)$$

with E_i, E_f the initial and final total energies. The matrix elements E and O cannot, in general, be dealt with easily. Two relatively simple approximations are the ‘ closure ’ approximation, and the assumption of plane waves for the final two-neutron state. The first of those may be expected to be valid at high photon energies. The second neglects the interaction of the final two-neutron system, and will be least valid when the final neutrons have a low relative velocity.

For high photon energies it might be expected that the struck proton would absorb all the momentum transferred. This is confirmed by the fact that the integrals E and O have a strong maximum for $\mathbf{P}=\frac{1}{2}\mathbf{q}$. The δ-function in (37) then just describes the free proton production kinematics. Removing the δ-function from the integrals and using the closure properties

$$d\sigma=(2\pi)^{-2}\,d\mathbf{k}[\{|\mathbf{K}|^2+|L|^2\}-\{\tfrac{1}{3}|\mathbf{K}|^2+|L|^2\}V], \qquad . \quad . \qquad (38)$$

where the first term gives the free proton production, and the second gives an interference term with

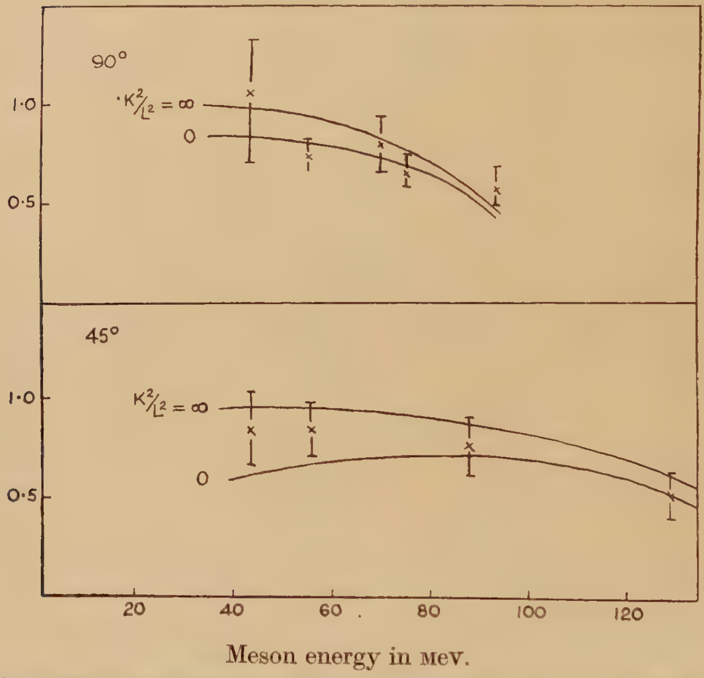
$$V = \int \cos(\mathbf{q}\cdot\mathbf{r})[v(r)]^2\,d\mathbf{r}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (39)$$

V decreases rapidly as $|q|$ increases so that for a given ν the interference is important only at small angles.

The plane wave approximation seems more likely to be applicable to existing experiments. The details of the calculation are elementary but lengthy, and are given by Lax and Feshbach (1952) and Saito *et al.* (1952). For a given quantum energy they lead to a differential cross-section

$$\frac{d\sigma}{d\Omega_k\,dk_0} = (2\pi)^{-2}kk_0[(|\mathbf{K}|^2+|L|^2)I_1-(\tfrac{1}{3}|\mathbf{K}|^2+|L|^2)I_2], \quad . \qquad (40)$$

Fig. 7



Ratio of π^+-photo-production in deuterium and hydrogen at 45° and 90° (laboratory system).

where I_1, I_2, are defined in the paper quoted. If the assumption is made that the ratio $|\mathbf{K}|^2/|L|^2$ remains constant over the spectrum of incident photons responsible for mesons observed in a given experiment, then for a given bremsstrahlung beam the ratio of the cross sections for deuterium and hydrogen can be expressed in terms of the known hydrogen excitation curve. This has been done by White *et al.* (1952). Their results, drawn for the extreme cases of $K^2/L^2=\infty$ and $K^2/L^2=0$, are shown in fig. 7 compared with the measured values.

The agreement is quite satisfactory, but it is scarcely discriminatory as to the value of $|K/L|$.

The production of charged mesons at heavier nuclei has been considered theoretically by Butler (1952), following earlier work by Lax and Feshbach (1951). The experimental results have already been reviewed, and the $A^{2/3}$ variation of the total yield of charged mesons noted. This result seems to denote that only nucleons on the surface of the nucleus contribute to meson production. The first suggestion is that mesons produced in the interior of the nucleus are absorbed before they emerge. Whilst in fact there is a considerable amount of absorption of this sort the evidence of meson scattering experiments suggests that light and medium nuclei show a degree of transparency to 60 MEV mesons (suggestive of a mean free path around $7{\cdot}0\times10^{-13}$ cm in nuclear matter) which would make this form of absorption insufficient. Another possibility, discussed by Wilson (1952) is that ‘ internal ’ absorption of the meson by, say, a pair of nucleons before it is actually produced, will be a strong competing process, leading to nuclear photodisintegration. This second process will clearly only apply to nucleons in the core where the nucleon interaction is strong. Thus it will also favour surface production.

In his calculations Butler (1952) considers only surface production, expanding the initial wave function into sums over products of single particle wave functions for a single proton and states of the remaining nucleus. In the conditions of the experiment of Littauer and Walker (1952) he finds

$$\sigma \geqslant Z\sigma_f\tau$$

where τ denotes the fraction of a time a typical proton spends outside the core and σ_f is the free proton-meson production cross section. An estimate for τ is $\tau=(1+\frac{2}{3}\kappa r_0)^{-1}$, where r_0 is the radius of the core, and κ is the nucleon momentum corresponding to an energy of 8 MEV. In general $\kappa r_0>1$ and, as r_0 varies like $A^{1/3}$, the $A^{2/3}$ variation of σ is immediate. Butler is able to show that the surface yield will contribute a substantial proportion of that observed by Littauer and Walker. Equally the general trend of π^-/π^+ ratios is in accord with the idea that for any particularly loosely bound particles (such as the odd neutron in ^{9}Be), for which τ approaches unity, production becomes large. The falling π^-/π^+ production ratio for symmetrical nuclei also accords with the increased Coulomb repulsion leading to lower binding energies for the protons.

The photo-production of neutral mesons in deuterium has recently been studied theoretically by Francis (1952). His calculation is in form a non-relativistic weak coupling one. However, the interaction matrix elements considered are varied so as to secure the possibility of agreement with experiment. Four interactions are tried :—

(i) The full non-relativistic interaction, including interaction with the proton magnetic moment γ_p

$$H_1=\frac{f}{\mu}(\boldsymbol{\sigma}\,.\,\nabla)\phi-\frac{e\gamma_p}{2M}\,\boldsymbol{\sigma}\,.\,\mathrm{curl}\,\mathbf{A}-e(\mathbf{p}\,.\,\mathbf{A})/M-\frac{e}{M\mu}(\boldsymbol{\sigma}\,.\,\mathbf{A})\phi.$$

(ii) Same as the above but without the direct production term.

(iii) Interaction of the radiation field only with the anomalous proton moment

$$H_3=\frac{f'}{\mu}(\boldsymbol{\sigma}\cdot\nabla)\phi-\frac{e\gamma_p'}{2M}\boldsymbol{\sigma}\cdot\operatorname{curl}\mathbf{A}.$$

(iv) Interaction chosen to give distribution $(2+3\sin^2\theta)$ to the π^0-mesons produced; for production of meson momentum $\mathbf{k}'$ in the c.m. system by a quantum energy $\boldsymbol{\nu}'$, polarization direction $\boldsymbol{\lambda}'$

$$H_4=\frac{ef''}{M\mu^2}\frac{2\pi}{\sqrt{(\nu'\omega')}}\exp[i(\boldsymbol{\nu}'-\mathbf{k}')\cdot\mathbf{r}][2\mathbf{k}'\cdot(\boldsymbol{\nu}'\times\boldsymbol{\lambda}')+i\boldsymbol{\sigma}\cdot\{\mathbf{k}'\times(\boldsymbol{\nu}'\times\boldsymbol{\lambda}')\}].$$

The elastic production of π^0-mesons is first calculated, i.e. in the process $\gamma+d\rightarrow\pi^0+d$. As pointed out by Francis and Marshak (1952) and Heckrotte, Henrich and Lepore (1952) this process is very sensitive to the sign of the neutral meson coupling to neutron and proton. It becomes very small indeed if $f_n=f_p$, but if, as in symmetric theory, $f_n=-f_p$ then for small angles it is large, and comparable indeed to the total π^0-production cross section including the continuous nuclear final states. In this region also the momentum transfer $\boldsymbol{\nu}-\mathbf{k}$ is relatively small, so only relatively low momentum components of the deuteron wave-function are involved and the calculation becomes more reliable.

The total production (including inelastic processes) is also considered for comparison with the experimental result at larger angles. The experimental ratio of approximately two for the deuterium to the hydrogen yield at 90° suggests that interactions of the form H_3 or H_4, approximately equal for neutrons and protons, are required.

§ 4. MESON NUCLEAR SCATTERING

4.1. *Meson Scattering by Nucleons*

In some respects the scattering of mesons by nucleons may be considered the most fundamental of all mesonic processes, analogous as it is to the Compton scattering of photons. Perhaps significantly too in no other problem has the ineffectiveness of weak coupling approximations been more apparent. The recent experimental evidence on the processes has been obtained principally at Chicago by Anderson, Fermi and their collaborators—Anderson *et al.* (1952 a, b) and Fermi *et al.* (1952)—and at Columbia—Chedester *et al.* (1951), Isaacs *et al.* (1952). For scattering at protons there are three processes to be considered :—

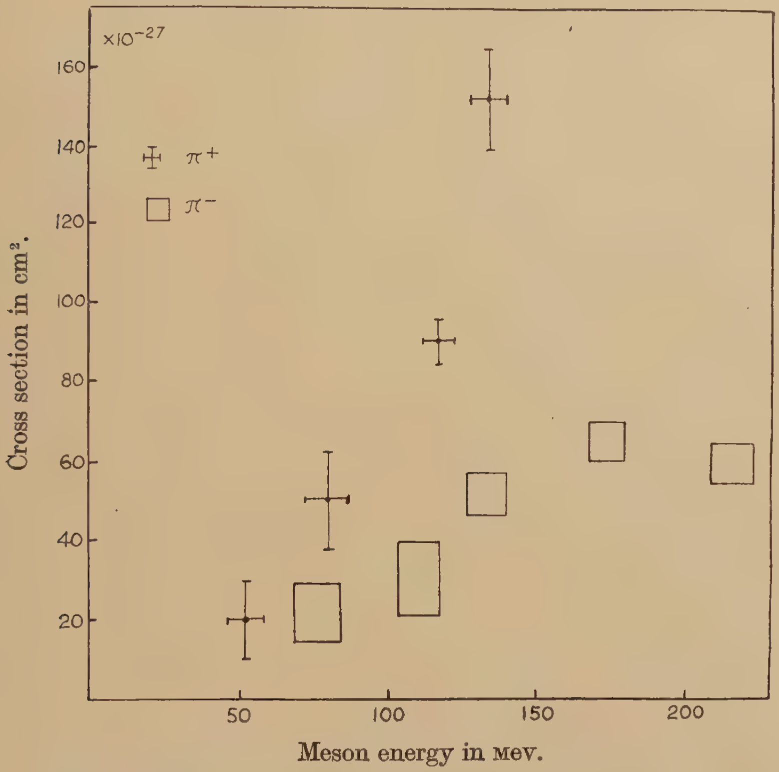
(i) $\pi^++p\rightarrow\pi^++p$,

(ii) $\pi^-+p\rightarrow\pi^-+p$,

(iii) $\pi^-+p\rightarrow\pi^0+n\rightarrow 2\gamma+n$.

The third is the so-called charge exchange scattering. By arguments of detailed balance from photo-production of mesons it may easily be shown that the radiative absorption process $\pi^-+p\rightarrow n+\gamma$ has a much smaller cross section than the others (about 1 or 2 m.b. in the energy region of present experiments). In fig. 8 the variation with energy of the total cross sections (i)–(iii) is shown: in this figure the two negative meson processes are added together. At Chicago a separation has also been made of the contribution of 'ordinary' and 'charge exchange' processes to the negative meson scattering. At meson energy 118 Mev it was roughly estimated that the charge exchange scattering was twice as large as the ordinary.

Fig. 8



Total scattering cross sections for π^+- and π^--mesons in hydrogen.

More recent results from Chicago have dealt with the angular distribution of the scattering. The elastic scattering of positive π-mesons of 110 Mev and 135 Mev has been measured at laboratory angles of 45°, 90°, 135°. The results converted to c.m. system angles are shown in table 7.

Table 7. Differential Cross Sections (c.m. system) for π^+–p Scattering

	$d\sigma(\pi^+, \pi^+)/d\Omega$ in 10^{-27} cm²/sterad.	
$\theta_{\text{c.m.}}$	110 Mev	135 Mev
56°	$3{\cdot}2 \pm 0{\cdot}6$	$5{\cdot}5 \pm 2$
104°	$5{\cdot}0 \pm 0{\cdot}6$	$6{\cdot}5 \pm 2$
145°	$11{\cdot}9 \pm 0{\cdot}9$	$20{\cdot}9 \pm 3$

It will be seen that backward scattering predominates. With the assumption that only meson s and p states contribute to the scattering the results may be expressed in the form (in the c.m. system)

$$\frac{d\sigma}{d\Omega}=a+b\cos\theta+c\cos^2\theta. \qquad (41)$$

The values of the coefficients a, b, c are shown in table 8.

Table 8. Coefficients for Angular Distribution of $(\pi^{\pm}$, p) Scattering

Meson energy (MEV)	Process	a	b (in $10^{-27}\mathrm{cm}^2$/sterad.)	c
110	(π^+, π^+)	$3{\cdot}5\pm0{\cdot}6$	$-4{\cdot}6\pm0{\cdot}8$	$7{\cdot}2\pm1{\cdot}8$
135	(π^+, π^+)	$3{\cdot}8\pm2{\cdot}2$	$-6{\cdot}8\pm2{\cdot}7$	$17{\cdot}5\pm6{\cdot}6$
135	(π^-, π^-)	$1{\cdot}2\pm0{\cdot}2$	$-0{\cdot}1\pm0{\cdot}3$	$0{\cdot}3\pm0{\cdot}7$
135	(π^-, π^0)	$1{\cdot}1\pm0{\cdot}6$	$-2{\cdot}5\pm0{\cdot}5$	$6{\cdot}3\pm1{\cdot}9$

In table 8 the results of π^--scattering at 135 MeV are also included. The (π^-, π^0) cross sections have to be obtained from the measured cross section $\sigma(\pi^-, \gamma)$ by the method of § 2.1. Whilst backward scattering again predominates for the (π^-, π^0) process, it is of interest that the $(\pi^-; \pi^-)$ process is nearly isotropic.

As has already been stated, these results are completely at variance with the predictions of weak coupling theory. The most complete relativistic treatment of this problem has been given by Ashkin, Simon and Marshak (1950), also by Peshkin (1951). Confining attention to pseudoscalar meson theories one finds that pseudoscalar coupling yields cross sections for processes (i) and (iii) which decrease with increasing meson energy. Corrections of order g^4 have also been worked out for the matrix elements in this case, but, in view of the large coupling constant required ($g^2/4\pi\simeq7$) it appears certain that no final theoretical judgment can be made on the basis of a weak coupling approximation. The experimentally observed increase of cross section suggests, however, in this approach, the use of a derivative coupling. In lowest order, and neglecting nucleon recoil, one then obtains for the differential cross sections in the c.m. system

$$\left.\begin{aligned} d\sigma(\pi^+,\pi^+)&=\left(\frac{f^2}{4\pi}\right)^2\frac{4k^4}{\mu^2\omega^2}\frac{d\Omega}{\mu^2},\\ d\sigma(\pi^-,\pi^-)&=d\sigma(\pi^+,\pi^+),\\ d\sigma(\pi^-,\pi^0)&=2\cos^2\theta\, d\sigma(\pi^+,\pi^+). \end{aligned}\right\} \qquad (42)$$

Striking differences are found between these weak gradient coupling results and experiment. Theory would predict a ratio 5/3 for total π^-/π^+ scattering, where experiment around 150 MeV shows nearer 1/3. Equally the angular distributions observed are quite unexplained. The inclusion of relativistic terms does nothing to help agreement with experiment, one of its main effects indeed being to produce forward scattering in the c.m. system for the (π^+, π^+) process.

In these circumstances it has seemed wise to abandon any immediate effort to secure agreement with the experimental results starting from a postulated interaction Hamiltonian between the nucleon and meson fields, but rather to concentrate attention on the insight that can be obtained by a partial wave analysis of the scattering data, along the lines more familiar from the treatment of nuclear collisions. Such an analysis can be helped by the use of the concept of isotopic spin. If attention is confined to the symmetric theory, in which the isotopic spin is a constant of the motion, a considerable simplification is thus achieved. The total isotopic spin is defined as $\mathbf{T}=\mathbf{L}+\frac{1}{2}\boldsymbol{\tau}$, where $\mathbf{L}$ denotes the meson isotopic operator, and $\boldsymbol{\tau}$ that for the nucleon. The convention is used that $L_3=+1$ corresponds with a π^--meson, and $\tau_3=+1$ corresponds with a neutron. The two permissible values of T are $T=1/2$ and $T=3/2$. Under the assumption of charge symmetry there will be no mixing of these two values. A typical eigen-state of isotopic spin will be denoted, say, $T^{3/2}_{1/2}$ corresponding to $T=3/2$, $T_3=1/2$. In terms of these eigen-states the typical meson–nucleon charge combinations can be expanded as follows:

$$\left.\begin{aligned}(\pi^+,\ \mathrm{p})&=T^{3/2}_{-3/2},\\ (\pi^-,\ \mathrm{p})&=\frac{1}{\sqrt{3}}\{T^{3/2}_{1/2}+\sqrt{2}\,T^{1/2}_{1/2}\},\\ (\pi^0,\ \mathrm{n})&=\frac{1}{\sqrt{3}}\{\sqrt{2}T^{3/2}_{1/2}-T^{1/2}_{1/2}\}.\end{aligned}\right\}\quad . \ . \ . \ . \ . \ . \quad (43)$$

The transition matrices for the different forms of π-nucleon scattering can then be written in terms of those between the isotopic states:

$$\left.\begin{aligned}(\pi^+\mathrm{p}\,|\,\pi^+\mathrm{p})&=(T^{3/2}\,|\,T^{3/2}),\\ (\pi^-\mathrm{p}\,|\,\pi^-\mathrm{p})&=\tfrac{1}{3}[(T^{3/2}\,|\,T^{3/2})+2(T^{1/2}\,|\,T^{1/2})],\\ (\pi^0\mathrm{n}\,|\,\pi^-\mathrm{p})&=\frac{\sqrt{2}}{3}[(T^{3/2}\,|\,T^{3/2})-(T^{1/2}\,|\,T^{1/2})].\end{aligned}\right\}\quad . \ . \quad (44)$$

In terms of these expansions it is very suggestive that scattering only through the $T=3/2$ states would give a ratio 9 : 1 : 2 for the three cross sections, in reasonable agreement with the experimental results. An analysis of this sort was given by Brueckner (1952), Nambu and Yamaguchi (1951). Brueckner considered only the interaction of p-wave mesons, and thus had only four states to consider: $\mathrm{P}_{1/2}$, $\mathrm{P}_{3/2}$ with $T=1/2$, $3/2$. In accordance with the predictions of strong coupling theory the lowest isobar will be a $J=3/2$, $T=3/2$ state. Brueckner postulated the existence of such an isobar at about 137 MEV excitation (corresponding to 200 MEV mesons in the laboratory system). Scattering would then occur predominantly through this state.

Brueckner's model, on the whole, however, appears to be rather a detailed one (apart from the uncertainties involved in the use of the Breit–Wigner formula, and perhaps even more so in the use of the weak couplin approximation to fix the relative contributions of the various non-resonant processes). Anderson, Fermi and others (1952) have given a straightforward phase analysis of their results, based on symmetric meson theory, and assuming that only s- and p-meson waves are involved. There are thus six phases to be determined at each meson energy, to fit nine experimental points. The most recent experimental data are well fitted by the choice (reported at Rochester Conference, Dec. 1952),

		$S_{1/2}$	$P_{1/2}$	$P_{3/2}$
135 MeV	$T=3/2$	16·1°	−6·1°	−41·8°
	$T=1/2$	−11·1°	5·1°	1·2°
120 MeV	$T=3/2$	17·8°	−4·1°	−31·6°
	$T=1/2$	−10·2°	3·1°	0·3°

There is a consistent uncertainty of sign in all these phase shifts. The general expressions for the different cross sections for arbitrary phase shifts can easily be evaluated and are given for reference.

$$k^2\, d\sigma(\pi^+, \pi^+)/d\Omega = |A_0+(2A^+_1+A^-_1)\cos\theta|^2 + |A^+_1-A^-_1|^2 \sin^2\theta, \qquad (45)$$

$$k^2\, d\sigma(\pi^-, \pi^-)/d\Omega = \tfrac{1}{9}\{|A_0+2B_0+(2A^+_1+A^-_1+4B^+_1+2B^-_1)\cos\theta|^2 + |A^+_1-A^-_1+2B^+_1-2B^-_1|^2 \sin^2\theta\}, \quad . \quad . \quad . \quad . \quad (46)$$

$$k^2\, d\sigma(\pi^-, \pi^0)/d\Omega = \tfrac{2}{9}\{|A_0-B_0+(2A^+_1+A^-_1-2B^+_1-B^-_1)\cos\theta|^2 + |A^+_1-A^-_1-B^+_1+B^-_1|^2 \sin^2\theta\}. \quad . \quad . \quad . \quad . \quad . \quad (47)$$

Here A, B denote $[\exp(2i\delta)-1]/2i$ respectively for scattering from the $T=3/2$, $T=1/2$ states, A_0, B_0 referring to s-wave scattering, A^+_1, A^-_1 referring to p-wave scattering with $J=3/2$ and $1/2$ respectively, and similarly for B^+_1, B^-_1. δ is the phase shift.

The phase shifts found by Fermi and collaborators do give some support to the idea of a predominating interaction through the $T=3/2$, $J=3/2$ state. Interactions are, however, also seen to be strong in the $T=1/2$, $S_{1/2}$ state and the $T=3/2$, $S_{1/2}$. Unfortunately, also, the ascription of phases is not up to the present unique, and Yang* has shown that an equally good fit to the Chicago data can be found with the dominant interactions the $T=3/2$, $P_{1/2}$ and $T=3/2$, $S_{1/2}$ ones. It should be added that the success of the phase shift analysis in fitting the experimental data on the assumption of the charge independence hypothesis is the strongest evidence as yet in support of this hypothesis.

Only very brief reference will be made here to the possibility of a strong coupling treatment of the meson scattering problem. For pseudoscalar mesons (PV coupling) a quantum mechanical strong coupling treatment has been given by Pauli and Dancoff (1942). A classical strong coupling treatment for neutral mesons is given by Pauli (1946), and has recently been extended by Feldman (1952) to the charge symmetric meson field. The strong coupling theory is described by two parameters, the source radius a and the isobar spacing energy ω_0, say, where in terms of the 'ordinary coupling' constant $\omega_0=3a\mu^2/2f^2$. The strong coupling conditions are

$$1/a \gg \mu, \quad \omega_0 \ll \mu. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (48)$$

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* Reported by Pais at the Copenhagen Conference, June 1952.

The isobar spacings are then given by

$$E=[J(J+1)-\tfrac{3}{4}]\omega_0. \qquad (49)$$

A typical result for scattering is that given by Pauli (1946) for neutral pseudoscalar mesons

$$\sigma=\frac{32}{9}\pi\frac{f^4}{\mu^4}\frac{k^4}{\omega^2}\frac{1+|R|^2}{|1-R^2|^2}, \qquad (50)$$

where $R=\xi+i\eta$ with $\xi=(\omega^2-a\mu^3)/\omega\omega_0$, $\eta=ak^3/\omega\omega_0$. Provided $a\mu\ll 1$, $a\omega_0\ll 1$ one can write

$$|1-R^2|^2\simeq(1-\omega/\omega_0)^2+4(ak^3/\omega_0{}^2)^2,$$

and the scattering cross section will have a resonance at $\omega=\omega_0$, with half width $ak_0{}^3/\omega_0$. In this way behaviour corresponding with the experimental total cross section can be simulated with $\omega_0\simeq 1{\cdot}8\,\mu$. This value lies outside the range of the strong coupling approximation, and the results should, therefore, be taken only as suggestive. It would seem, however, that such classical treatments form a better first approximation to the meson–nucleon scattering problem than is provided by perturbation theory. The calculations of Feldman (1952) show also that the ratio of $\sigma(\pi^-, \pi^0)$ to the sum of $\sigma(\pi^+, \pi^+)$ and $\sigma(\pi^-, \pi^-)$ for scattering on protons is reasonably given by the classical model. In conclusion it may be noted that the strong coupling treatment neglects the s-wave meson–nucleon interaction. The general deficiencies of strong coupling theory, which preclude it ever being accepted as a reasonable complete description of meson phenomena, are discussed in more detail in the accompanying paper by Moorhouse.

An interesting test of the s-wave meson–nucleon interaction has been suggested by Anderson and Fermi (1952). From the s-wave scattering $\pi^-+\mathrm{p}\rightarrow\pi^0+\mathrm{n}$ it should be possible to deduce the 'charge exchange' rate of capture of π^--mesons from the K-shell in hydrogen. This capture rate is not directly known, but from the measurements of Panofsky, Aamodt and Hadley (1951) it is known to be comparable with the radiative capture rate $\pi^-+\mathrm{p}\rightarrow\mathrm{n}+\gamma$. The rate for this last process can be estimated from its inverse, so that the two capture processes can be compared. For this purpose Anderson and Fermi used the s-wave phases established to hold at 135 or 110 MEV. More recent measurements at Chicago (quoted by Marshak 1952) suggest for the s-phases in the $T=3/2$ state, $\delta=\pm 21°$ at 135 MEV, $\pm 13°$ at 113 MEV, $\pm 6°$ at 78 MEV, so that the s-phase variation has not yet become linear with the wavelength at the lowest energies of experiment. At sufficiently low energies, however, a scattering length may be defined by $a=-\delta/k$. Only the $T=3/2$ state will be considered, so that

$$\sigma(\pi^-\mathrm{p}, \pi^0\mathrm{n})=\frac{8\pi}{9}a^2\frac{v_0}{v}, \qquad (51)$$

where v, v_0 denote the relative velocities of the incoming π^- and outgoing

π^0 with respect to the nucleon. Equally if the cross section for the process $\gamma + \mathrm{n} \rightarrow \mathrm{p} + \pi^-$ is denoted by σ_γ, then

$$c(\pi^- + \mathrm{p} \rightarrow \mathrm{n} + \gamma) = \frac{k^2}{2{p_\gamma}^2}\sigma_\gamma . \qquad (52)$$

The two cross sections (51) and (52) should be approximately equal, and hence a is determined in terms of known quantities.

The rapid variation of the s-wave phase referred to above indicates that this phase may change sign at an energy of about 40–50 MeV. It has been noted by Marshak (1952) that such a change of sign would be expected if the effective nucleon–meson potential had a repulsive core surrounded by an attractive region. At low energies only the attraction would be experienced, giving positive phases: at increased energies the core would be penetrated and the phase become negative. This idea has points in common with a more field theoretical approach recently made by Drell and Henley (1952). These authors use the pseudoscalar meson theory with pseudoscalar coupling which has the advantage of being renormalizable. In its original form this theory is quite unsuitable for use with weak coupling methods. However, the transformation suggested by Dyson (1948) to demonstrate the partial equivalence of the (PS) and (PV) coupling can be applied (cf. § 1.3 above). The transformed Hamiltonian contains, in addition to the (PV) coupling term for meson–nucleon interaction, with $f=(\mu/2M)g$ in the usual notation, and higher order terms, the importance of which it is difficult to assess, a large ' core ' term of the form

$$H_c = M\int \psi^+\gamma_4\left[\left(1+\frac{ig}{M}\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\varphi}\right)\exp\left\{-\frac{ig}{M}\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\varphi}\right\}-1\right]\psi\, d\mathbf{r}. \qquad (53)$$

In lowest order this term yields

$$\frac{g^2}{2M}\int\bar{\psi}\psi\phi^2\, d\mathbf{r}.$$

If the nucleons can be treated as fixed sources, so that for a typical nucleon one can replace $\bar{\psi}\psi$ by $K(r-r_n)$ then approximately

$$H_c = \frac{g^2}{2M}\int K(r-r_n)\phi^2\, d\mathbf{r},$$

giving a short range repulsive meson–nucleon interaction. Higher order corrections may be shown to be equivalent to multiplying H_c by a factor $\lambda(<1)$. Since the major part of the s-wave scattering comes from this term the effect is to increase the relative importance of the p-wave scattering, and obtain cross sections increasing with energy in conformity with the experiment.

The theory involves three parameters, the coupling strength f(or g), the source radius a, and the height of the repulsive potential specified by λ. A typical set of values chosen by Drell and Henley is $a=1/M$, $\lambda=0{\cdot}075$, $g^2/4\pi=70$. A distorted wave approximation taking full account of H_c is used, and the radiation damping equation of Heitler and Wilson (1941) employed. Agreement is not yet found with experiment, but at least a considerable improvement is shown.

The work of Drell and Henley has been referred to in some detail in view of its use of the (PS) coupling with pseudoscalar meson theory. This form of coupling has recently shown some promise in the treatment of nuclear forces (described in the accompanying article by Moorhouse). It has the great advantage that a consistent renormalizable theory can be constructed. Hitherto, however, it has not been displayed to any advantage in calculations of meson production and scattering. This was inevitable because, so far as they are not completely phenomenological, these calculations have been carried out either on a basis of weak coupling, utterly unjustifiable for the large pseudoscalar coupling constant, or of strong coupling, which, by its essentially non-relativistic nature, is applicable only to pseudo-vector coupling. Any suggestion, therefore, of a treatment applicable to pseudoscalar coupling is of considerable interest. There may otherwise be a suspicion that the attractiveness of this theory is to some extent based on the fact that so few results can be deduced from it with any semblance of validity.

4.2. *Scattering of π-Mesons at Deuterium*

While the experiments on the scattering of π-mesons in hydrogen and their analysis have thrown great light on the π-meson–proton interaction no such direct method exists for detecting the π-meson–neutron interaction. General arguments of charge symmetry indeed make it most probable that the cross section for (π^-, n) scattering will, apart from Coulomb effects, be exactly equal to that for (π^+, p), and similarly for (π^+, n) and (π^-, p). However, it is of great interest to establish whether these corresponding neutron and proton scattered waves have amplitudes of equal or opposite sign, apart from the importance of directly verifying the requirements of charge symmetry. Experiments towards this end have been carried out at Columbia by Isaacs *et al.* (1952 b) and at Chicago by Anderson *et al.* (1952).

The experiments of Isaacs *et al.* (1952) were carried out with 60 MeV mesons with results as below:

$$\left.\begin{aligned}\sigma(\pi^+,\text{p})&=27{\cdot}8\pm2{\cdot}5\text{ m.b.},\\ \sigma(\pi^-,\text{p})&=17{\cdot}6\pm2{\cdot}2\text{ m.b.},\end{aligned}\right\}\text{giving}$$

$$\sigma(\pi^+,\text{p})+\sigma(\pi^-,\text{p})=45\pm3{\cdot}5\text{ m.b.},$$

$$\sigma(\pi^+,\text{d})=38{\cdot}3\pm3{\cdot}1\text{ m.b.},$$

$$\sigma(\pi^-,\text{d})=32{\cdot}8\pm3{\cdot}1\text{ m.b.}$$

The near equality of the (π^+, d), (π^-, d) cross sections, within experimental error, is confirmation of the expected charge-symmetry. To establish the size of possible interference effects from the neutron and proton scattered meson waves it is necessary to subtract from $\sigma(\pi^+$, d) about 7 m.b. corresponding to the absorption of π^+-mesons by the process $\pi^+ + \mathrm{d} \rightarrow \mathrm{p} + \mathrm{p}$. Thus for scattering, charge exchange and radiative absorption processes one finds $\sigma^{\mathrm{sc}}(\pi^+, \mathrm{d}) \simeq 30$ m.b. compared with 45 m.b. from the sum of the n and p cross sections (with the assumption that $\sigma(\pi^-, \mathrm{p}) = \sigma(\pi^+, \mathrm{n})$). This seems to indicate quite a considerable destructive interference of the meson waves scattered from neutron and proton. If the amplitude of the waves are taken from perturbation theory one finds that such destructive interference is expected for scalar mesons or pseudoscalar mesons, (PV) coupling. However PS (PS) mesons would be strongly constructive. This result can scarcely be taken very seriously in view of the need, already pointed out, for the suppression of the s-wave interaction in the last-named theory.

The Chicago results—Anderson *et al.* (1952)—generally show the same destructive interference effect, though, in some cases, the difference is not so pronounced. Results are given for a wide range of meson energies up to 200 MEV for π^- and 130 MEV for π^+. For example, for 135 MEV π^- mesons the results are,

$$\sigma(\pi^-, \mathrm{d}) - \sigma(\pi^-, \mathrm{p}) = 76 \pm 15 \text{ m.b.},$$

$$\sigma(\pi^+, \mathrm{p})\{= \sigma(\pi^-, \mathrm{n})\} = 135 \pm 15 \text{ m.b.}$$

An absorption correction has still to be made to the first cross section, so in this case the interference has again reduced the (π^-, d) cross section by over 30%.

A detailed theoretical analysis can be carried out along the lines of the impulse approximation. Investigations of this sort on a weak coupling basis have been given by Cheston (1952) and Segall (1951) and on a phenomenological basis by Fernbach, Green and Watson (1952) and Isaacs *et al.* (1952 b). If the radiative absorption, which is small, is ignored, and the ordinary and charge-exchange scattering amplitudes for positive mesons are written, in the notation of the last-named authors,

$$I_s = s + \boldsymbol{\sigma} \,.\, \mathbf{S}, \qquad I_c = c + \boldsymbol{\sigma} \,.\, \mathbf{C}, \quad . \quad . \quad . \quad . \quad . \quad (54)$$

normalized so that the free cross sections are

$$\frac{d\sigma(\pi^+, \mathrm{p})}{d\Omega} = |s^{\mathrm{p}}|^2 + |\mathbf{S}^{\mathrm{p}}|^2, \qquad \frac{d\sigma(\pi^+\mathrm{n}, \pi^0\mathrm{p})}{d\Omega} = |c^{\mathrm{n}}|^2 + |\mathbf{C}^{\mathrm{n}}|^2, \quad . \quad (55)$$

then with the assumption of the validity of a ‘ closure ’ approximation

$$\frac{d\sigma(\pi^+, \mathrm{d})}{d\Omega} = \frac{d\sigma(\pi^+, \mathrm{p})}{d\Omega} + \frac{d\sigma(\pi^+, \mathrm{n})}{d\Omega} + F(\varDelta\mathbf{k})\{2\,Re(s^{\mathrm{p}*}s^{\mathrm{n}})$$

$$+ \tfrac{2}{3} Re(\mathbf{S}^{\mathrm{p}*} \,.\, \mathbf{S}^{\mathrm{n}}) - |c^{\mathrm{n}}|^2 - \tfrac{1}{3}|\mathbf{C}^{\mathrm{n}}|^2\}, \quad . \quad . \quad (56)$$

where $\varDelta\mathbf{k}$ is the difference in the initial and final meson momenta, and F is a form factor for the deuteron

$$F(\mathbf{q}) = \int v_d^2(r) \exp(i\mathbf{q} \,.\, \mathbf{r})\, d\mathbf{r},$$

with v_d the deuteron wave-function.

On the whole it seems unlikely that an interference formula of the type of (56) can account for the experimental results given above. One possible explanation of the discrepancy would be the breakdown of the impulse approximation. The validity of the approximation in this context has been considered by Chew and Wick (1952) and Segall (1951). Essentially it rests on the smallness of the collision time τ, which may be defined as the reciprocal of the violation of energy conservation in the intermediate state. On a weak coupling picture the approximation will always be justified, but on a strong coupling picture with isobaric levels of a sufficiently narrow width it might break down. Calculations on these lines have been carried out by Miyazawa and Matsuyama (1952) who, by including higher order terms, find values of $\sigma_d/(\sigma_p+\sigma_n)$ nearer to those of experiment. They may, however, have chosen a rather narrow level half-width ($\simeq$ 30 Mev) for the spin 3/2 proton isobar.

4.3. *Meson Scattering at Nuclei*

With the availability of π-meson beams in several laboratories a considerable number of investigations on the scattering and absorption of π-mesons by nuclei have been carried out. A particularly interesting case, that of the deuteron, has already been treated separately. In this section only a rather incomplete review of the data available on other nuclei will be provided.

Among the most comprehensive experiments are those carried out on the scattering of π-mesons by Carbon. The most recent such experiment is by Byfield, Kessler and Lederman (1952). Other investigations are reported by Isaacs, Sachs and Steinberger (1952) and Shapiro (1952 a). Byfield *et al.* carried out their experiments using a cloud chamber containing a 2·14 g/cm^2 scattering carbon plate. The mesons had a mean energy of about 60 Mev, and the cross sections found are listed in table 9.

Table 9. π–C Cross Sections (in 10^{-27} cm^2)

	Stars				Elastic		Inelastic
	Number protons >15 Mev				20°–180°	65°–18°	$\Delta E>15$ Mev
	0	1	2	Total			
π^-	110	34	4	148 ± 18	165 ± 15	63	46 ± 10
π^+	34	77	42	153 ± 22	89 ± 10	38	15 ± 8

The sum of all the cross sections measured is about 350 m.b. for π^-, 260 m.b. for π^+, which compares with a geometrical cross section of 360 m.b. for ^{12}C (using $R=1{\cdot}47A^{1/3}\times10^{-13}$ cm and making no allowance for the meson wavelength). It is clear, therefore, that Carbon shows a considerable degree of transparency to 60 Mev π-mesons. Earlier results of the same kind were found by Shapiro (1952) using mesons with an average energy about 48 Mev. Shapiro's data were analysed by Bethe and Wilson (1951), using the optical model of Fernbach, Serber and Taylor (1949). In this model the nucleus is assigned a refractive index, defined as the ratio of the meson wave number inside the nucleus to that outside i.e. $k+k_1/k$, and an absorption index K. If the B.K.W. approximation is valid then the meson wave propagates with a variation

$\exp\{i(k+k_1)z-\frac{1}{2}Kz\}$ inside the nucleus. Using this approximation the 'shadow scattering' or diffraction scattering cross section σ_d, and the absorption cross section σ_a are derived in terms of k_1, k and the nuclear radius R. Here the absorption cross section can, in general, be taken to mean all scattering other than diffraction scattering. Given the opacity of the nucleus, i.e. $\sigma_a/\pi R^2$, K is immediately determined and hence the reaction mean free path, λ say. If the diffraction cross section is also known then k_1 can be found. Instead of k_1 one may substitute the average potential energy of the meson in the nucleus, V_0 say. The results are rather sensitive to the choice of nuclear radius, but using $R=1{\cdot}47A^{1/3}\times 10^{-13}$ cm one finds

$$\lambda \simeq 4{\cdot}3\times 10^{-13}\ \text{cm}, \qquad V_0\simeq 15\ \text{Mev}.$$

The use of the optical approximation is not in fact very well justified in this case, and in the analysis of their results Byfield *et al.* have preferred to use the partial wave solution for a complex scattering potential $V_0-i\sigma_0$. This is determined so as to yield the observed mean nuclear elastic scattering, after elimination of Coulomb scattering, and mean total absorption cross section. With the same choice of R as above $V_0=18$ Mev, $\sigma_0=9$ Mev. These results correspond to a mean free path in nuclear matter of about 8×10^{-13} cm.

Other points of especial interest in the results of Byfield *et al.* are :—

(i) Interference between Coulomb and nuclear scattering shows up clearly in the angular distribution curves for elastic scattering. The sign of this interference establishes that on the potential model above V_0 must be an attractive potential.

(ii) In the absorption of π^+-mesons some of the stars (about 12%) show two fast emergent protons consistent with the reaction $\pi^+ + \text{d} \rightarrow \text{p}+\text{p}$. When allowance is made for the fraction of such protons that would make nuclear collisions before emergence, it appears that a considerable fraction of the π^+-absorption must be due to this process. Brueckner, Serber and Watson (1951) had already suggested such a mechanism in a theoretical analysis of absorption processes for mesons. Quantitatively one finds with this model, making use of the known cross section for the process, that the nucleus is equivalent to about $5Z$ deuterons. This is not surprising as it is the high relative momentum components of the n–p pair that are concerned in the reaction, and these will be much stronger in the closely packed nucleus than in the relatively loose structure of deuterium.

An alternative analysis of the π-meson scattering data in Carbon has been given by Peaslee (1952). The scattering is assumed to be the sum of single nucleon resonance scatterings, only the s- and p-waves for such scattering being included. If the nucleons are imagined in a single particle orbit specified by a wave-function $\phi(\mathbf{r})$ then the amplitude for scattering of the meson from an initial momentum state $\mathbf{k}_i$ to a final state $\mathbf{k}_f$ will contain a factor

$$\int \phi^* \exp(i\Delta\mathbf{k}\,.\,\mathbf{r})\phi\, d\mathbf{r}=\rho(\Delta\mathbf{k}), \quad . \quad . \quad . \quad . \quad . \quad (57)$$

where $\Delta\mathbf{k}=\mathbf{k}_i-\mathbf{k}$. On summing the amplitudes from the different nucleons the coherent scattering will be the largest, and with reasonable choice of s- and p-wave interactions it proves possible to fit the experimental data very well. However, this explanation of all the elastic scattering in terms of only the coherent resonance scattering of the separate nucleons takes no account of the fact that considerable meson absorption is simultaneously taking place. In the complete absence of resonance elastic scattering this absorption would itself provide diffraction scattering, which may form quite a considerable part of the elastic scattering observed. It would, therefore, appear necessary to make simultaneous allowance for the absorption process.

Many other experiments have added to the accumulating store of data on meson-nuclear scattering. Among these may be mentioned the experiments of Bernardini and his collaborators (cf. Bernardini and Levy 1951) on scattering and absorption in photographic emulsions, and by Martin (1952) on scattering of 113 and 137 MeV mesons in various elements. At the higher energies nuclei become almost completely opaque, and the diffraction scattering can be obtained from the hard-sphere model. The almost total absence of charge exchange scattering $\pi^-+(Z, A)\to\pi^0+(Z-1, A)$ is also confirmed.

§ 5. MESON PRODUCTION IN NUCLEAR COLLISIONS

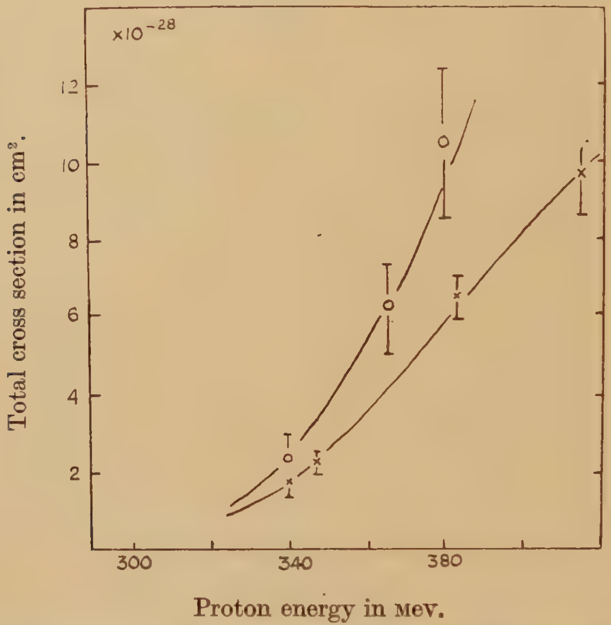
5.1. *Meson Production in Nucleon–Nucleon Collisions*

Experiments on the production of π-mesons in hydrogen by 340 MeV protons were first carried out at Berkeley by Peterson, Cartwright and their collaborators. There have been two main series of experiments, one using liquid hydrogen as a target, and the other using a subtraction method (measuring the carbon-polyethylene difference). In both cases the mesons were detected in photographic emulsions. The first set of experiments are reported by Peterson (1950), Peterson *et al.* (1951 a, b). The second set are due to Cartwright *et al.* (1950, 1951) and Whitehead and Richman (1951 a, b). One of the most striking effects observed in these experiments is the peaking of the meson energy spectrum, measured at any angle towards the highest available energy. Subsequent more accurate measurements have indeed lead to the conclusion that a substantial fraction of the mesons—a half or more at the energies concerned—are produced in the reaction $p+p\to\pi^++d$. Coincidence deuterons have also been observed by Crawford *et al.* (1951), but not in such numbers as to give a quantitative estimate of their frequency. In addition to the proof that the process $p+p\to\pi^++d$ is responsible for much of the meson production, these measurements also suggest that the continuum of mesons is strongly peaked towards high energies.

Apart from the ratio of the continuum to the discrete cross sections the work of Peterson, Cartwright *et al.* is of interest as determining the angular variation of the integrated cross section. Measurements at 0° and 60° in the laboratory system of the mesons in the peak (presumed associated with $p+p\rightarrow\pi^{+}+d$) give for the differential cross section $1{\cdot}3\pm0{\cdot}26\times10^{-28}$ cm²/sterad and 8×10^{-30} cm²/sterad respectively under 341 MeV proton bombardment. These suggest an angular distribution in the c.m. system $(3{\cdot}2\pm0{\cdot}8)\times(0{\cdot}07\pm0{\cdot}07+\cos^2\theta)\times10^{-29}$ cm²/sterad, giving $\sigma=1{\cdot}6\pm0{\cdot}5\times10^{-28}$ cm².

The Peterson *et al.* measurements were made with liquid hydrogen at 18°, 30° 64°. They confirm the $\cos^2\theta$ distribution of the mesons in the c.m. system. Recently also excitation curves for this reaction have been measured at Berkeley and Columbia. The Columbia measurements are briefly described by Passman, Block and Havens (1952 b).* Measurements are made at 90° in the laboratory system—a region in which the differential cross section has only 1/10th of its value in the forward direction. Measurements were made with incident proton energies of 345, 365 and 380 MeV. Resolution was not good enough to distinguish the continuum and discrete production. For the total cross section the

Fig. 9



Total cross section for π^{+}-production in p–p collisions against the proton energy in the laboratory system.

(*a*) including transitions to continuous final states (estimated from 90° laboratory measurements, assuming a $\cos^2\theta$ distribution).

(*b*) for process $p+p\rightarrow n^{+}+d$ (deduced from the inverse reaction).

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* I am indebted to Dr. Passman for a copy of his thesis containing a full description of this work.

excitation curve of fig. 9 was found. The curve has an approximate variation $\propto T^2$, where T is the meson kinetic energy in the c.m. system.

By contrast the measurements of Schulz *et al.* (1952) suggest a $T^{3/2}$ variation in the lower energy region up to 340 MeV, though this measurement applies to mesons in the high energy peak and hence may be appropriate to the $p+p \rightarrow \pi^+ + d$ process.

These production experiments can be looked at in conjunction with the experiment of Durbin, Loar and Steinberger (1951 a) and Clark, Roberts and Wilson (1951) on the inverse reaction $\pi^+ + d \rightarrow p+p$. Again the $\cos^2\theta$ angular distribution is approximately verified. The latest results of Durbin *et al.* (1951 b) give:

Table 10

π^+ Energy MeV	$\frac{d\sigma}{d\Omega}(\pi^+ + d \rightarrow p+p)$	Total cross section
25	$9(\cos^2\theta + 0{\cdot}22) \times 10^{-28}$ cm²/sterad	$3{\cdot}1 \pm 0{\cdot}3 \times 10^{-27}$ cm²
40	$18(\cos^2\theta + 0{\cdot}2)$	$6{\cdot}1 \pm 0{\cdot}6$
53	$21{\cdot}5(\cos^2\theta + 0{\cdot}18)$	$7{\cdot}0 \pm 0{\cdot}7$

In fig. 9 these results are inverted using detailed balance (with spin 0 mesons) to give the expected variation with proton energy of the total cross section for the $p+p \rightarrow \pi^+ + d$ reaction.

5.2. *Theory of Production in* p–p *Collisions*

It may be expected that the theoretical predictions regarding the production of mesons in nucleon collisions will be even less reliable than those for photon production. The former is, with a field theoretic treatment and on the assumption that nuclear forces are due to π-meson exchange, a third order process in the mesonic coupling, where the latter is only of first order. Such third order calculations have been carried through by Morette (1949) and Brueckner (1950). They will, however, be very unreliable in predicting the production of mesons in p–p collisions at the sort of energies at which experiments have up to now been carried out, say up to 500 MeV, for in this region the results are very critical to the detailed interaction of the final 2-nucleon system, which is poorly described by perturbation theory. A more hopeful approach is to use a phenomenological description of the inter-nucleon interactions in the initial and final state. The process then becomes a first order one in the meson coupling constant. Calculations of this sort were first carried out by Foldy and Marshak (1949), who however still confined themselves to a perturbation treatment of the initial and final 'distorted wave' states. With this method it is in fact important to give as accurate an account as possible of the initial and final nucleon states. The first treatment of this sort was given by Gunn, Power and Touschek (1951 a and b). Similar treatments were later given by Fujimoto and Yamaguchi (1951) and Cheston (1951), the latter dealing mainly with the inverse reaction $\pi^+ + d \rightarrow p+p$.

There are three likely sources of error in calculations of this type which we may call the phenomenological-weak coupling approximation.

(i) Large momentum transfers are involved in meson production by p–p collisions. The calculation is thus dependent, say, in the case $p+p\rightarrow\pi^{+}+d$ on a knowledge of the high momentum components of the deuteron wave function which may not be well described by a wave function, such as the Hulthen one adopted by most of the authors referred to above.

(ii) Only a weak coupling treatment is given to the meson–nucleon interaction. The possible errors arising from this step are very considerable as may be seen from the failure of the weak coupling approximation in π-nucleon scattering.

(iii) It is not clear that the method of ‘ distorted waves ’ can validly be employed to treat the emission of π-mesons which are themselves responsible for the nuclear field.

In discussion of these errors a treatment of (iii) has been given by Power (1951), which at least forms a partial justification for the method. The error involved in (i) is hard to assess. The Hulthen wave function in fact gives a momentum distribution $\propto p^{-4}$ at high momenta which may fall off rather too slowly. However, the differential partial waves might be expected to be similarly affected by this error which can, therefore, scarcely account for the wide discrepancies between the theoretical results found by the method and experiment. It seems likely that (ii) is in fact the principal source of error. If, in the spirit of the isobaric states approximation, which appears useful in accounting for π-nucleon scattering phenomena, one is willing to assume that π–p interactions only proceed strongly through certain isobaric states, then results very different from those of weak coupling theory will be obtained (e.g. Matsuyama and Miyazawa 1952).

An alternative to the weak coupling approach is the purely phenomenological one adopted by Watson and Brueckner (1951). In this a form is assumed for the transition matrix $(F|R|I)$ describing the meson production process, subject only to the necessary conditions of relativistic invariance. The initial and final states are decomposed into partial waves just as for the weak coupling treatment. Watson and Brueckner assume a rather restricted form of R in so far as the nucleon transition is concerned. For the meson emission, however, they list a wide variety of operators, and one can be found giving results in approximate agreement with experiment.

Weak Coupling Treatment

The weak coupling treatment of Gunn, Power and Touschek (1951) will first be briefly outlined for the case of pseudoscalar mesons (PV) coupling. The gradient coupling is chosen as more suited to a weak coupling treatment. Even if the actual matrix elements predicted by this treatment are at fault the formalism can be generalized by the substitution of phenomenological interactions with the correct transformation properties. With the notation described above the matrix element for π^+-meson production in p–p collisions is

$$H'_{FI}=-\frac{if}{\mu}(2\omega)^{-1/2}\int d\mathbf{r}\,\psi_F{}^{\dagger}\{(\boldsymbol{\sigma}^{(1)}\,.\,\mathbf{k}-\omega\rho_1{}^{(1)})\exp\left(-\tfrac{1}{2}i\mathbf{k}\,.\,\mathbf{r}\right)$$

$$\mp(\boldsymbol{\sigma}^{(2)}\,.\,\mathbf{k}-\omega\rho_1{}^{(2)})\exp\left(\tfrac{1}{2}i\mathbf{k}\,.\,\mathbf{r}\right)\}\psi_0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (58)$$

where the — sign holds for a final charge singlet state,
+ sign holds for a final charge triplet state.

ψ_F, ψ_0 are wave functions for the final neutron–proton and the initial di-proton states respectively, and $\mathbf{r}$ is the relative coordinate of the two nucleons. In (58) the ρ_1 terms give the 'odd' part of the interaction allowing transitions between nucleon states of different parity. A non-relativistic approximation has to be made for these. Attention will be confined to the case where the final nucleon system is the deuteron in its ground state, with wave function v_d.

The approximation is made by G.P.T. that the nucleon interaction vanishes in the odd parity (triplet spin) initial states. The matrix element for transitions from these states is

$$H'_{FI}=\frac{if}{\mu}(2\omega)^{-1/2}(3\sigma_{m_s'}\,|\,\boldsymbol{\sigma}^{(1)}\,.\,\boldsymbol{\sigma}^{(2)}\,|^3\,\sigma_{m_s})\,.\,[\mathbf{k}\{I(\mathbf{p}_0+\tfrac{1}{2}\mathbf{k})-I(\mathbf{p}_0-\tfrac{1}{2}\mathbf{k})\}$$

$$+\frac{\omega}{M}\mathbf{p}_0\{I(\mathbf{p}_0+\tfrac{1}{2}\mathbf{k})+I(\mathbf{p}_0-\tfrac{1}{2}\mathbf{k})\}], \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (59)$$

where $$I(\mathbf{p}_0)=\int\exp\left(i\mathbf{p}_0\,.\,\mathbf{r}\right)v_d\,d\mathbf{r}.$$

Here, particularly near the threshold, the second term dominates. It leads to an approximately isotropic meson distribution with a dominant meson s-wave from the ^{3}P–^{3}S transition. This second term is due to the 'odd' part of the interaction Hamiltonian, and, to give agreement with the experimental result that mesons are produced dominantly in a $\cos^2\theta$ angular distribution, the assumption is often made that this interaction is suppressed. This assumption has to be watched in view of the π^-+d absorption which proceeds exactly through the process here disallowed, i.e. an s-wave meson is absorbed by the deuteron in its ^{3}S ground state to form two neutrons in a ^{3}P state.

For the moment we proceed on the assumption that the most important contribution comes from the initial singlet spin (even parity) states. With the omission of the 'odd' term and neglect of the meson momentum the matrix element becomes

$$H'_{FI}=-\frac{if}{\mu}(2\omega)^{-1/2}(3\sigma_{m_s}\,|\,\boldsymbol{\sigma}^{(1)}\,.\,\boldsymbol{\sigma}^{(2)}\,|\,^1\sigma_0)\,.\,\mathbf{k}\int\psi_F{}^{\dagger}\psi_0\,d\mathbf{r}. \quad . \quad . \quad (60)$$

In the treatment of G.P.T. only the ^{1}S initial two-proton and the ^{3}S part of the deuteron state were considered. The mesons are then distributed in isotropic p-waves, and the overlap integral depends on the difference between the singlet and triplet potentials—by orthogonality properties it would become zero if these two were the same.

The detailed calculations of G.P.T. were carried out using a Hulthen wave-function for the deuteron ground state

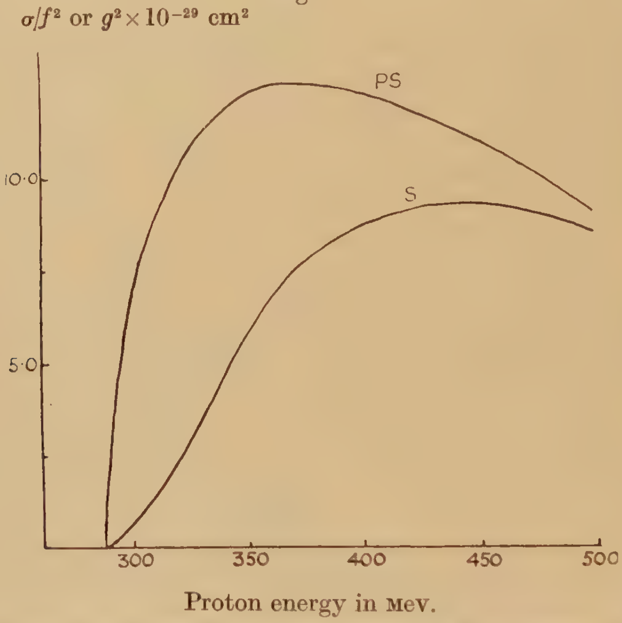
$$v_d(r)=\left\{\frac{\kappa b(b^2-1)}{8\pi}\right\}^{1/2}\frac{1}{r}\exp\{-\tfrac{1}{2}(b-1)\kappa r\}[1-\exp(-\kappa r)], \quad . \quad (61)$$

with $b=MJ/\kappa^2$, where the potential $U=J\exp(-\kappa r)/[1-\exp(-\kappa r)]$ and $1/\kappa=1{\cdot}17\times10^{-13}$ cm, ${}^3J=46{\cdot}6$ Mev, ${}^1J=27{\cdot}2$ Mev. The final expression for the cross section (including all terms) is

$$\frac{d\sigma}{d\Omega}=\frac{f^2}{v\mu^2}\frac{b^3(b^2-1)}{\left\{1+\frac{(b+1)^2\kappa^2}{4p_0^2}\right\}^2}\frac{k^3\kappa^5}{p_0^8}\left\{\left(\frac{\Delta J}{J}\right)^2+2\left(\frac{\omega p_0}{kM}\right)^2-\frac{8\omega}{M}\cos^2\theta\right\}. \quad (62)$$

where f^2 denotes the square of the coupling constant in ‘ ordinary charged ’ units (cf. § 1·3), and p_0 is the initial proton momentum in the c.m. system.

Fig. 10



Theoretical excitation curves for π^+-production in the reaction $p+p\rightarrow\pi^++d$ for scalar and pseudoscalar mesons.

In fig. 10 the energy variation of the total cross section predicted by the weak coupling pseudoscalar theory is shown. This is very different from the experimental result shown in fig. 9.

It happens in fact that scalar meson theory would in this approximation have secured apparently better results. In place of (58) one would now have

$$H_{FI}=g(2\omega)^{-1/2}\int d\mathbf{r}\,\psi_F^{\dagger}\{\exp(\tfrac{1}{2}\,.\,\mathbf{k}\,.\,\mathbf{r})\mp\exp(-\tfrac{1}{2}\,.\,\mathbf{k}\,.\,\mathbf{r})\}, \quad . \quad (63)$$

where again the $\mp$ signs correspond with the final isotopic singlet and triplet states. No spin change is permitted, so the dominant transition may be expected to be that from the ^{3}P initial di-proton, yielding a $\cos^2\theta$ meson distribution. For transitions to the bound deuteron state

$$\frac{d\sigma}{d\Omega}=\frac{12g^2}{v}\frac{b^3(b^2-1)}{\left\{1+\frac{(b+1)^2\kappa^2}{4p_0^{\,2}}\right\}^2}\frac{k^3\kappa^5}{p_0^{\,10}}\cos^2\theta. \quad . \quad . \quad . \quad . \quad (64)$$

This cross section is also shown in fig. 10. The intial excitation curve varies with $T_\pi^{3/2}$, but this rise is reduced, effectively by the $1/p^4$ variation of the high frequency Fourier components of the deuteron ground state wave-function, and is not so steep as experiment requires.

The mesons produced by transition to the continuous n–p state are also of interest. The theoretically predicted curves are of the same general form as those of fig. 10, but the maximum occurs at higher energies. At energies above 400 MeV theory indicates that these mesons will become more numerous than those produced in transitions to the final deuteron state. The experimental results (cf. fig. 9) again suggest the absence of the s-mesons allowed by the pseudoscalar theory, which would both lead to a relatively rapid initial rise of the cross section, and further result in a meson energy spectrum approximately uniform in the c.m. system, contrary to experiment.

5.3. *Other Nucleon–Meson Production Reactions*

A reaction of considerable interest but not yet studied experimentally* is $n+p\rightarrow\pi^0+d$. A distinction must be made between ‘ neutral ’ and ‘ symmetric ’ theory—in the former equal signs of coupling constant hold between meson and neutrons and protons ; in the latter the τ_3 changes the sign for these two cases. This has the result that for transitions to the final deuteron $^1\tau_0$ ground state the initial state must be $^3\tau_0$ for the symmetric theory, $^1\tau_0$ for the neutral theory. Hence for the symmetric theory the allowed initial states for the n–p system are ^{1}S,^{3} P, ^{1}D etc. just as for the reaction $p+p\rightarrow\pi^++d$. The matrix elements are indeed identical with those for this reaction—the only difference arising from the normalization of the incident wave. One finds

$$\sigma(n+p\rightarrow\pi^0+d)=\tfrac{1}{2}\sigma(p+p\rightarrow\pi^++d). \quad . \quad . \quad . \quad . \quad (65)$$

For neutral (or τ_4) coupling, on the other hand, the permissible initial states are ^{3}S, ^{1}P, ^{3}D etc. It might be expected that p- and s-wave mesons would be emitted respectively in transitions from the first two states. However, if the ‘ odd ’ part of the interaction is ineffective these transitions are disallowed, and d-wave mesons are the lowest to be given off. In the absence of a definite model, giving satisfactory results for the $p+p\rightarrow\pi^++d$ process, predictions regarding neutral meson production must also be treated with caution, but it seems most likely that ‘ neutral ’ theory neutral mesons will give an angular distribution and cross section quite different from that determined for symmetric theory by (65). The $n+p \rightarrow \pi^0+d$ reaction, if it were experimentally practicable, would therefore prove valuable in providing definite evidence on the character of neutral meson coupling, cf. Power (1952).

---

* *Note added in proof.* A $\cos^2\theta$ distribution for this reaction has recently been reported by Hildebrand.

Consideration of the other main reactions $n+p \rightarrow \pi^+ + 2n$ and $p+p \rightarrow \pi^0+2p$ will be deferred until after the description of the phenomenological treatment of meson production by nucleons.

5.4. *Phenomenological Treatments*

The most thorough-going phenomenological treatment of meson production in nucleon–nucleon collisions is that of Watson and Brueckner (1951). The cross section is expressed in terms of the transition matrix R, $d\sigma=(2\pi)^4(dJ/v)\Sigma(F\,|\,R\,|\,I)^2$, with appropriate normalization of the initial and final state wave functions. Here dJ is the final accessible phase space volume.

The assumption of charge symmetric theory leads to interesting relationships between meson production in the different two-nucleon reactions. The total isotopic spin of the meson–nucleon system is in this case a constant of the motion, say $\mathbf{J}=\mathbf{T}+\mathbf{L}$, where $\mathbf{T}=\frac{1}{2}(\boldsymbol{\tau}_1+\boldsymbol{\tau}_2)$ denotes the resultant isotopic spin for the two nucleons, and $\mathbf{L}$ is that for the meson. The possible sub-states for the nucleon field are given by

$$
\begin{array}{lll}
T=1, & T_3=1 & \text{(2n) denoted say by } t^+, \\
 & T_3=0 & \text{(np) denoted say by } t^0, \\
 & T_3=-1 & \text{(2p) denoted say by } t^-, \\
T=0, & T_3=0 & \text{(np) denoted say by } s.
\end{array}
$$

For the meson the states $L_3=1, 0, -1$, corresponding to π^-, π^0, π^+ mesons respectively, will be denoted m^+, m^0, m^-, so that the total charge is $(1-J_3)$. The possible states of the total isotopic spin are then $J=2, 1, 0$ $(T=1)$ and $J=1$ $(T=0)$. Of these the $J=2$ state is forbidden in nucleon–nucleon meson production since the initial 2-nucleon state has $J=0$ or 1. The possible final isotopic spin states of the two nucleon+meson system are

$$\underline{J=1\ (T=1)} \qquad\qquad \underline{J=0\ (T=1)}$$

$$|1,1\rangle=\frac{1}{\sqrt{2}}(m^0t^+-m^+t^0) \qquad |0,0\rangle=\frac{1}{\sqrt{3}}(t^-m^+-t^0m^0+t^+m^-)$$

$$|1,0\rangle=\frac{1}{\sqrt{2}}(m^-t^+-m^+t^-)$$

$$|1,-1\rangle=\frac{1}{\sqrt{2}}(m^0t^--m^-t^0)$$

$$\underline{J=1\ (T=0)}:\ sm^+,\ sm^0,\ sm^-.$$

The possible initial states are

$$\underline{J=1}\quad t^+,\ t^0,\ t^-. \qquad \underline{J=0},\quad s.$$

There are thus only three independent matrix elements for which the following equalities are derived.

Initial Isotopic Singlet

$$\langle t^- m^+ | R | s\rangle = -\langle t^0 m^0 | R | s\rangle = \langle t^+ m^- | R | s\rangle = M_2, \text{ say.} \qquad (66)$$

Initial Triplet (final triplet of nucleon isotopic spin)

$$\langle m^0 t^+ | R | t^+\rangle = -\langle m^+ t^0 | R | t^+\rangle = \langle m^- t^+ | R | t^0\rangle = -\langle m^+ t^- | R | t^0\rangle$$
$$= -\langle m^- t^0 | R | t^-\rangle = \langle m^0 t^- | R | t^-\rangle = M_1. \quad . \quad . \quad . \quad . \qquad (67)$$

Initial Triplet (final singlet of nucleon isotopic spin)

$$\langle s m^+ | R | t^+\rangle = \langle s m^0 | R | t^0\rangle = \langle s m^- | R | t^-\rangle = M_3. \quad . \quad . \qquad (68)$$

These are all the permitted transitions. Other transitions permitted by charge conservation are $\langle m^0 t^0 | R | t^0\rangle$ and $\langle m^0 s | R | s\rangle$, both of which are totally forbidden.

For various nucleon–nucleon collisions it is now possible to draw up a list of the meson producing transitions with their isotopic dependences. This is given in table 11 from Watson and Brueckner, where only final nucleon s-states are considered.

Table 11. Classification of Nucleon Transitions Producing s- and p-wave Mesons

	Meson in s-state		Meson in p-state	
pp, π^+ nn, π^-	${}^3P_0 \rightarrow {}^1S_0$ ${}^3P_1 \rightarrow {}^3S_1$	M_1 M_3	$\left.\begin{matrix}{}^1S_0\\{}^1D_2\end{matrix}\right\}\rightarrow {}^3S_1$	M_3
pp, π^0 nn, π^0	${}^3P_0 \rightarrow {}^1S_0$	M_1	forbidden	
np, π^+ np, π^-	${}^3P_0 \rightarrow {}^1S_0$	M_1	$\left.\begin{matrix}{}^3S_1\\{}^3D_1\end{matrix}\right\}\rightarrow {}^1S_0$	M_2
np, π^0	${}^3P_1 \rightarrow {}^3S_1$	M_3	$\left.\begin{matrix}{}^3S_1\\{}^3D_1\end{matrix}\right\}\rightarrow {}^1S_0$	M_2
			$\left.\begin{matrix}{}^1S_0\\{}^1D_2\end{matrix}\right\}\rightarrow {}^3S_1$	M_3

It is interesting now to review all the evidence on meson production in nucleon–nucleon collisions in the light of table 11. The experimental evidence has already been reviewed for the process $p+p \rightarrow \pi^+ + d$. There are two other basic reactions to be considered.

(*a*) $p+p \rightarrow \pi^0 + 2p$. With 340 MeV protons the cross section for this process is believed to be very small, certainly less than one-twentieth of the cross section for $p+p \rightarrow \pi^+ + (np)$ and perhaps very much smaller. With 430 MeV protons it is greatly increased to about $0{\cdot}4 \times 10^{-27}$ cm^2, which is about one-sixth of the extrapolated (pp, π^+) yield at this energy. These measurements are more fully discussed below.

(*b*) $n+p \rightarrow \pi^+ + 2n$ *or* $\pi^- + 2p$. Charge symmetry suggests that, apart from small Coulomb effects, these two reactions should be equal. Up to the present, evidence is only available from bombardment of complex nuclei, and is discussed more fully below. A very striking result is shown in deuterium, where, at 90° in the laboratory system about 25 times as many π^+ as π^- are found with 380 MeV proton bombardment. This effect has never been thoroughly analysed, but may be partially explained by exclusion principle and angular distribution effects. The general trend of production experiments in complex nuclei suggests that these reactions while not so large as the (pp, π^+) are not an order of magnitude smaller.

Earlier arguments have suggested that the ‘ odd ’ operators in pseudoscalar theory should be strongly suppressed. Nucleon state parity changes would then be forbidden, and, near the threshold, mesons would be predominantly emitted in p-states. This is certainly in agreement with observation in several respects.

(i) The small (pp, π^0) cross section is explained. This cross section, however, would be small for many different hypotheses, e.g. if π-mesons were scalar a rough estimate along the lines of the weak coupling calculations would make it about one-thirtieth of the (pp, π^+) cross section at 350 MEV.

(ii) The (np, π^+) cross sections contain a different matrix element M_2 and p-wave mesons are allowed, so that the reaction can be as strong as required to give agreement with experiment.

(iii) The $\cos^2\theta$ distribution of (pp, π^+) mesons becomes explicable. Special arguments are still necessary to secure this distribution, which will clearly *not* result from the ${}^1S_0 \rightarrow {}^3S_1$ transition. For example, Watson and Brueckner suggest an expression of the form $(\boldsymbol{\sigma}\,.\,\mathbf{p})(\mathbf{k}\,.\,\mathbf{p})$ in the transition matrix element.

We may contrast this suggestion with the proposal of Chew *et al.* (1951) in consideration of the inverse process. Inverting their argument to the $p+p \rightarrow \pi^+ + d$ case, the ‘ odd ’ interactions are again supposed to be suppressed, so that the important transitions are those from the initial even parity ${}^1S$, ${}^1D$ states. The ${}^3D$ part of the final deuteron state is, however, not ignored. The matrix element for the transition is then proportional to

$$\int \psi_d{}^\dagger (\boldsymbol{\sigma}^{(1)} - \boldsymbol{\sigma}^{(2)})\,.\,\mathbf{k}\psi_0\, d\mathbf{r}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (69)$$

Introducing the phase shifts δ_0, δ_2 of the initial proton S and D waves Chew *et al.* find an angular distribution of the form

$$F_0{}^2 + \sqrt{2}\cos(\delta_0 - \delta_2) F_0 F_2 (3\cos^2\theta - 1) + \tfrac{1}{2} F_2{}^2 (3\cos^2\theta + 1), \quad . \quad (70)$$

where $$F_0 = \int_0^\infty u_0(r) u_d(r)\, dr, \qquad F_2 = \int_0^\infty w_0(r) w_d(r)\, dr.$$

u_d, w_d are the S and D radial parts of the deuteron wave functions, and u_0, w_0 similarly for the initial two-proton system. The angular distribution will become anisotropic if $F_2 > F_0$. This might at first sight seem improbable, but it must be remembered that u_0, u_d will be nearly orthogonal if the singlet and triplet central forces are approximately equal. On the whole, however, the argument is rather a critical one.

Bethe and Austern (1952) have applied the arguments of Watson, Brueckner and Chew *et al.* to the (np, π^+) reaction. Here (cf. table 11) with the odd interaction suppressed, the ${}^3S_1 + {}^3D_1 \rightarrow {}^1S_0$ process will be the dominant one—the mesons coming out in P-states. The angular distribution of the mesons, however, will be very different on the two assumptions. The W–B interaction fairly directly again gives a distribution of the $\cos^2\theta$ type. The Chew treatment correlates the meson direction only with the nucleon spins, and anisotropy can exist only to the extent caused by the tensor coupling of the $^3S_1$ to the $^3D_1$ initial states—the latter D state being of course oriented with respect to the nucleon relative momentum. With reasonable guesses for the quantities involved Bethe and Austern estimate only a 20% $\cos^2\theta$ contribution to the otherwise isotropic distribution.

There are too many uncertainties both regarding the validity of the theories and too many gaps in the experimental information available for very definite conclusions to be drawn from this analysis of the nucleon–nucleon meson production. It can at least be stated, however, that, presuming the mesons to be pseudoscalar, some modification of the weak-coupling transition elements is necessary to secure the $\cos^2\theta$ distribution of the reaction. Further experimental evidence on the (pp, π^0) and (np, $\pi^\pm$) reactions would be helpful in discriminating between the modifications proposed.

An alternative approach to an explanation of the $\cos^2\theta$ distribution is by means of an isobaric intermediate state of the kind already considered in meson-scattering and photo-production. Suggestions of this sort have been made by Matsuyama and Miyazawa (1952) and Brueckner and Watson (1952). The meson production now proceeds as a two-stage process—firstly, in the nuclear collision one of the nucleons is excited to its $j=3/2$ isobaric state ; secondly this nucleon decays with the emission of a meson. If the initial nucleon state is ^1D then the intermediate state is an S-state of spin 2, and the meson angular distribution is $(3\cos^2\theta+1)$ which is not too far out of agreement with experiment.

5.5. *Meson Production in Nucleon–Nucleus Collisions*

Particular interest attaches here to the measurement of the cross sections for meson production by the bombardment with protons of deuterium. This is the most elementary process from which information regarding the (pn, π^+) and (pn, π^0) cross sections can be obtained. Experimental information on the production of $\pi^\pm$-mesons has been provided by Passman, Block and Havens (1952 a). They made measurements at 90° in the laboratory system by taking the difference in $\pi^\pm$-mesons produced by bombardment of C and $(CD_2)_n$ targets with 381 ± 5 MEV protons. The π^+-spectrum produced had a broad maximum in the neighbourhood of 30–50 MEV, and an integrated cross section

$$d\sigma^+/d\Omega=2{\cdot}9\pm1{\cdot}2\times10^{-28}\ \text{cm}^2/\text{sterad}$$

(at 90° in laboratory system). The negative meson cross section was much smaller $d\sigma^-/d\Omega=1{\cdot}1\pm0{\cdot}9\times10^{-29}$ cm²/sterad.

These experiments can be analysed with the impulse approximation. This approximation would seem reasonable in view of the smallness of the collision time compared with the characteristic deuteron period. The further assumption has to be made, however, that in the final state there is still no interaction between the two colliding particles and the spectator. Particularly at high meson energies this is likely to be a rather inaccurate assumption. The extreme opposite would be to consider the possibility of the reactions $p+d\rightarrow\pi^{+}+{}^{3}H$ or $\pi^{0}+{}^{3}He$. No such reaction is possible with π^--production which might thus be much smaller than π^+-production. However, the experimental results do not seem to support this as a complete explanation of the large π^+/π^- ratio, as no pronounced high energy meson peak is observed. The legitimate conclusion appears to be that the (np, π^+) cross section is, in fact, considerably smaller than that of (pp, π^+) at the energies considered (<400 Mev), though it would be valuable to have this result confirmed at other angles. The theory of the 'elastic' reactions $p+d\rightarrow\pi^{+}+{}^{3}H$, or $\pi^{0}+{}^{3}He$ has been briefly considered by Ruderman (1952). Their ratio forms an interesting test of charge independence. By considerations similar to that of § 5.3 it can immediately be seen that the first process should have twice the cross section of the second. Ignoring the contribution from the p–n collision to the π^+-production, Ruderman estimates a cross section of $1{\cdot}3\times10^{-29}$ cm^2 for this process with 340 Mev protons.

Further evidence on the π^+/π^- ratio comes from the measurements on π^+, π^- production by protons at Carbon (Passman, Block and Havens 1952 c). These have been made at three energies. The π^+-production results are shown in table 12.

Table 12. Differential Cross Sections for π^+-production in Carbon

Incident proton energy (Mev)	345	365	380
$(d\sigma/d\Omega)$ 90° in 10^{-28}cm^2/sterad	$4{\cdot}7\pm1{\cdot}3$	$6{\cdot}4\pm1{\cdot}4$	$6{\cdot}7\pm1{\cdot}5$

π^--mesons are again produced in much smaller quantities. Two effects are, however, clear:

(*a*) The π^--meson spectrum is shifted towards lower energies than that of π^+-mesons,

(*b*) the π^+/π^- ratio is about $(10\pm3):1$.

This ratio (*b*) is much bigger than that quoted earlier by Richman and Wilcox (1950) who found $5{\cdot}1\pm1{\cdot}0$ for the same ratio for 340 Mev protons. However, it is in accord with the recent figure of Dudziak (1952) who finds a ratio of 15 : 1 at 0°. The similarity of these 0° and 90° figures suggest that the angular distributions of (np, π^-) and (pp, π^+) mesons are not dissimilar.

A large difference is shown between the energy distributions of the π^+- and π^--mesons from carbon at 0°. The π^- peak is at about 25 Mev, while the π^+ peak is above 70 MEV. The explanation of this may lie in the high Coulomb barrier against the low energy protons left when all the available energy is concentrated on a π^--meson.

An analysis of the π^+/π^- production ratio at 90° in carbon has been given by Passman and Block. They use the nucleon momentum distribution suggested by the ' quasi-elastic ' proton scattering experiments of Cladis, Hess and Moyer (1952)

$$N(p)=\text{const.}\exp(-p^2/\alpha^2), \quad . \quad . \quad . \quad . \quad . \quad (71)$$

where $\alpha^2/2M=14$–19 MEV. The Chew-Goldberger distribution

$$N(p)=C/(\alpha^2+p^2)^2$$

with α approximately the same as in (71) gives too large a high momentum spectrum.

As has been seen above, there is no reliable estimate of the (np, π^+) cross section, so the calculation is first carried out for the contribution from p–p collisions alone, in the impulse approximation. For the p–p production the reasonable estimates are made of

(i) $\cos^2\theta$ distribution of the π^+ in the c.m. system,

(ii) meson energy concentrated at the high energy end T_{max} in the c.m. system.

(iii) excitation function proportional to T^2_{max}.

No account need be taken of exclusion principle effects in the final state when meson emission at 90° in the laboratory system is considered since the final colliding nucleons will, in the laboratory system, have an energy above the filled carbon states.

The meson production can be calculated with the above approximations in a quite straightforward manner. The result has to be corrected for absorption of mesons within the nucleus before they are emitted. This can be done in the manner suggested by Brueckner, Serber and Watson (1951) on the basis of the optical nuclear model. In this way the fraction of the produced mesons actually emitted is calculated as

$$f=3\left\{\frac{1}{2x}-\frac{1}{x^3}+\frac{(1+x)\,e^{-x}}{x^3}\right\},$$

where $x=2R/\lambda_0$, $R=$nuclear radius$=a_0A^{1/3}$, $\lambda_0=$mean free path for absorption of mesons. For carbon the use of $\lambda_0=3a_0$ yields $f=0{\cdot}6$, though this value of λ_0 may be an underestimate.

The calculations yield meson energy spectra at 90° in good agreement with experiment. The increase in cross section in the interval 345–380 MEV is also well predicted. Of chief interest perhaps, however, is the absolute value of the cross section. In the energy interval studied, at 90°, the cross section is about 12 times that of hydrogen. Allowing for absorption (admittedly a rather uncertain factor) the meson production must be about 18 times that from hydrogen. The steep excitation function for the (pp, π^+) reaction does indicate some enhancement of the meson production at protons bound in carbon, viz. the increased cross section for protons with momenta towards the incident proton more than compensates for the decreased cross section for protons moving in the opposite direction. In this way Passman and Block calculate an enhancement factor 2. This would still leave 1/3 of the total cross section to be explained by (pn, π^+) processes.

There are considerable uncertainties in these calculations but it would appear that in this case the (pn, π^+) collisions are making a more effective contribution to meson production than the (pn, π^-).

A fair amount of information has been collected on the variation of meson production under proton (or neutron) bombardment with atomic number, and on the π^-/π^+ ratios. On the whole, however, this data does not form such a coherent picture as the corresponding data for photo-production. Clark (1952) has measured the yield of 40 MeV mesons produced by 240 MeV protons in various targets. π^- and π^+ yields were measured at different angles, 30°–50° and 130°–150° respectively, so that no absolute values of π^-/π^+ ratios can be given. The results are shown in table 13 where both cross sections are normalized to 1 for Be.

Table 13. Yields of 40 MeV π^+- and π^--mesons

Element	σ^+	σ^-	$\sigma^+/A^{2/3}$	$\sigma^-/A^{2/3}$
Be	1·00	1·00	1·00	1·00
C	3·52	1·74	2·9	1·4
Al	7·95	6·0	3·8	2·9
Cu	13·7	14·6	3·7	4·0
Ag	16·6	19·1	3·2	3·7
W	19·4	23·8	2·6	3·2
Pb	19·0	23·5	2·4	2·9

The yield appears to vary more rapidly than $A^{2/3}$ for light nuclei and more slowly for heavy nuclei. Similar measurements on π^+ production by 340 MeV protons have been made by Hamlin *et al.* (1951) with consistent results.

The theoretical analysis of this information is made very difficult by the lack of knowledge of the cross section for π^+-production in p–n collisions. Qualitatively, however, certain arguments may be applied, similar to those in the case of photo-production.

(i) Absorption of mesons produced in the nuclear core, supplemented by the absorption of the incident protons by scattering within the nucleus, may certainly reduce the yield to an $A^{2/3}$ variation or even slower.

(ii) Exclusion principle effects will apply very strongly to the π^--production in p–n collisions. If all the nucleons remain together the processes involved are :—

$$\begin{aligned} \mathrm{p}+(A, Z) &\to \pi^+ + (A+1, Z) \\ &\to \pi^- + (A+1, Z+2), \end{aligned}$$

the second of which has a considerable proton excess. Such effects will be strongest near the threshold, or where the meson takes most of the available energy.

(iii) The yields may equally be correlated with the variations in the binding energies of the final nucleon arrangements,

$$M(A+1, Z)-M(A+1, Z+2).$$

An analysis has been carried out by Henley (1952) making use of the impulse approximation. Neglecting interference terms, and assuming equal matrix elements for p–p and n–p production of π^+ one would find under proton bombardment $\sigma^+/\sigma^-=(A+Z)/(A-Z)$. This compares with an experimentally observed ratio of about 12 for protons between 340–380 MeV at carbon. Coulomb effects may, however, be strong in this case.

Neutral meson production from proton bombardment of light nuclei has been studied by Hales *et al.* (1952). Their results normalized to a yield of 6·0 from Carbon are shown in table 14.

Table 14. Relative Cross Section for π^0 Production by 341 MeV Protons

Nucleus	Relative cross section
^{1}H	0·07 ± 0·06
^{2}H	0·85 ± 0·20
^{9}Be	5·01 ± 0·12
^{10}B	4·99 ± 0·11
^{11}B	5·95 ± 0·13
^{12}C	6·00 ± 0·16
^{16}O	7·46 ± 0·28

It is seen that, to a large extent, the production may be assumed to proceed through the (np, π^0) process with no contribution corresponding to (pp, π^0). This has often been adduced as evidence from nucleon production of the pseudoscalar character of the π^0-meson. The argument has been discussed above and seems to be one of rather doubtful validity.

Recent evidence has also been presented for the higher energy production of π^0-mesons by 430 MeV protons on H and Be (Marshall *et al.* 1952). These results are very interesting as showing how at higher energies the selection rules precluding (pp, π^0) production are broken down. The differential cross section was measured in the forward direction for the production of the π^0 decay γ-rays. Two methods were used.

(i) The energy spectrum of the γ-rays was measured with a pair spectrometer.

(ii) The conversion electrons after the collimated γ-ray beam had passed through a lead converter were measured by a water Cerenkov counter. Only this method, in view of its greater efficiency, could be employed for the H measurements.

With the hydrogen target the forward γ-ray cross section was $d\sigma_\gamma/d\Omega=0{\cdot}20\times10^{-27}$ cm^2/sterad. Rather independently of the angular distribution of the π^0 emission this leads to $\sigma(\text{pp}, \pi^0)=0{\cdot}45\times10^{-27}$ cm^2

at 430 MEV. The result for ^{9}Be is similarly found to be $\sigma(\mathrm{p\ Be}, \pi^0)=8{\cdot}7\times10^{-27}\ \mathrm{cm}^2$. If it is assumed that $\sigma(\mathrm{Be})=4\sigma_{\mathrm{p}}+5\sigma_{\mathrm{n}}$, where σ_{p} is taken from the H results, then an estimate of $1{\cdot}35\times10^{-27}\ \mathrm{cm}^2$ is obtained for the (np, π^0) cross section. Approximately, this might be expected to be a little less than half of the (pp, π^+) cross section at the same energy. So large an extrapolation of the excitation curve for the latter reaction is no doubt dangerous, but fig. 9 would suggest that the relation is approximately fulfilled. A similar agreement is shown with the (p ^{12}C, π^0) cross section measured by Crandall for 340 MEV protons of $1{\cdot}4\times10^{-27}\ \mathrm{cm}^2$, which would suggest an (np, π^0) cross section about $2\times10^{-28}\ \mathrm{cm}^2$. These arguments, however, ignore the momentum distribution of the target nucleons which may considerably increase their effectiveness.

§ 6. CONCLUSIONS

It must be admitted at the conclusion of this article that up to the present the most striking contributions to the study of the production and scattering of mesons have been the experimental ones. The detailed investigations of meson scattering, of photo production of both charged and neutral mesons at protons, and of π^+ production in proton–proton collisions have all added considerably to appreciation of the fundamentals of the meson–nucleon interaction. In many directions there is room for more experimental work, even confining attention to the π-meson region, using accelerators below 500 MEV. Particularly obvious is the need for information on the production of both neutral and charged mesons from the bombardment of protons by neutrons, and of the various 'elastic' processes involving deuterium, e.g. $\gamma+\mathrm{d}\rightarrow\pi^0+\mathrm{d}$. However, none of the fundamental processes have yet been exhaustively studied, and the experimental errors are in some cases still large. Thus further details on the excitation curve and angular distribution near the threshold of the $\mathrm{p}+\mathrm{p}\rightarrow\pi^++\mathrm{d}$ reaction would enable more accurate deductions to be made regarding the relative importance of the meson s and p waves. Equally the difficult experiments on the angular distributions and excitation curves of photo-produced mesons, especially neutral mesons, are still rather incomplete. In all these experiments there are two aspects of principal interest. Firstly, near the threshold, where only a few partial waves can be concerned, the analysis of the results leads to very clear-cut conclusions regarding the degree to which the meson–nucleon system prefers to exist in certain states. Secondly, an idea of the validity of the conception of isobaric states and of the importance of radiation damping can be gained from the higher energy decrease of the cross sections. The higher energy machines now coming into use will greatly help in the clarification of these latter issues.

The theoretical conclusions of the greatest significance are those involving no detailed meson theory, but only the qualitative arguments of spin and parity. There are still many unexpected experimental results to be explained, but the general conclusion that the π-meson is pseudoscalar seems to rest on very strong evidence—the proof of zero spin from π^+ capture in deuterium and its inverse, and the proof of odd parity from the two-neutron absorption of π^-, again in deuterium. The pseudoscalar character of the π-meson is further supported in a general way by the requirements of nuclear forces, the spin dependence of which would be almost impossible to imagine on a purely scalar theory.

Accepting that the π-meson is pseudoscalar the line of theoretical advance is still not clear. In this article attention has been confined to lowest order perturbation theory and strong coupling methods, together with those more phenomenological approaches which do not attempt to trace observations back to a fundamental Hamiltonian, but only to render them consistent with broad assumptions on the meson character. It is probably unnecessary to review the theoretical progress separately in each of the three main fields studied. In each case the lowest order perturbation theory gives results in violent disagreement with experiment. In the two production cases further study has shown how by modifying the weak coupling interaction, adding or suppressing terms, the situation can be improved, but no fundamental principles guide these alterations, and the approach becomes largely a phenomenological one.

These arbitrary alterations, indeed express the acceptance of the indication from experiment that certain meson partial waves interact more strongly with nucleons that would be expected. It is then reasonable to take this conclusion to its extreme form in the assumption of the isobaric states suggested by strong coupling theory. There is a danger here that there will be so many adjustable parameters in such a theory, that, even if the idea is incorrect, a reasonable agreement with experiment can still be secured. Too much significance should, therefore, not yet be attached to the relative success of such methods, both in consideration of meson production and scattering. A simpler attitude is that the form of the excitation curve is influenced in its rise by the partial waves involved and the phase space available, and in its fall by conservation requirements, possibly supplemented by the effect of radiation damping which will be so formulated as automatically to include conservation.

In a general way there are at least three questions to which one would like to know the answers :—

(i) Are the interaction Hamiltonians of § 1 adequate to describe the phenomena, or, for example, must explicit non-linearities be introduced at the very outset.

(ii) Is the meson–nucleon coupling pseudoscalar or pseudo-vector.

(iii) Are the neutral and charged couplings related according to charge symmetry.

Of these questions (iii) at present seems to be nearest an answer. The π^+-proton scattering analysis certainly favours the charge independence hypothesis, and experiments, not yet performed, or only very incompletely reported, such as $n+p \rightarrow \pi^0+d$ or $\gamma+d \rightarrow \pi^0+d$ seem likely to resolve the remaining doubts in the near future. On the second question there is a theoretical prejudice towards the PS coupling in view of its renormalizability. Clearly it is a matter of great importance to seek better approximate methods of applying this coupling to the fundamental processes of meson production and scattering. Up to the present calculations are almost entirely confined to low order perturbation treatments which are quite unsatisfactory.

The first question is much more general, and it would seem that it can only be answered when an adequate mathematical technique is found for dealing with meson field problems based on the existing formalism. One obvious line for such an attack at present is the development and application of the methods of approximation and summation of the weak coupling series used by Levy in the treatment of nuclear forces, described in the accompanying article by Moorhouse. Methods such as that of intermediate coupling lead to tremendous complication of detail, and their accuracy is in the end rather questionable. Perhaps the greatest advance due to the work of recent years has been the elimination of many of the possible variants of meson field. Theoretical approach can now be concentrated on a single type of meson, and as a result, more rapid progress may be hoped for.

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